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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

VOLUME XXII DECEMBER, 1929 NUMBER 8

SOME OBJECTIVES TO BE REALIZED IN A COURSE IN PLANE GEOMETRY

BY SISTER ALICE IRENE

The College of St. Catherine, St. Paul, Minnesota

The title of this paper—"Some Objectives to be Realized in a Course in Plane Geometry"—naturally suggests to the mind of the reader the question: What Objectives? The National Council classifies Mathematical Objectives as (1) Practical or utilitarian, (2) Disciplinary, and (3) Cultural. Cultural Objectives are subdivided into three parts: (*a*) Appreciation of beauty in the geometric forms of nature, art, and industry, (*b*) Ideals of perfection as to logical structure, precision of statement and thought, discrimination between the true and the false, etc., and (*c*) Appreciation of the power of Mathematics. Important as are the practical and disciplinary ends, I am not going to touch on them except in an indirect way, and as to the cultural, I shall confine myself to only one division, namely, appreciation of beauty in the geometric forms of nature and art.

When the chairman of the committee approached me on the subject of this paper, he asked me to say something about what we are actually doing or have done in our school; I shall tell you, therefore, about an experiment which we tried out in the Derham Hall High School in the years 1926–27, 27–28 with the pupils of the tenth year. My choice of this particular experiment to speak about was actuated by the desire to tell to you what the cultivation of an appreciation of nature as seen through the geometric world has meant to us. I have divided my subject into three parts and I shall try to develop the topics in the following order: (1) The facts and conditions which led up to the experiment, (2) The procedure, and (3) The results.

I have been interested in the teaching of mathematics for up-

wards of ten years, and my experience has been confined almost entirely to schools for girls. The first few years were years of disappointment and worse, discouragement. Year after year I had to meet classes of tenth year girls, their number often increased by failures from the previous year. Practically all of these students were taking plane geometry because they had to. In this they were typical of the majority of high school girls everywhere. All their lives these girls have heard that mathematics is hard, uninteresting, and meant mainly for men, that women never have done very much in the field, and that most normal girls dread it; consequently, they are depressed and fearful of failure or at least of a low grade. The psychological effect of this attitude of mind is obvious. It is prophetic of failure. But what is worse, the idea that geometry is a subject of little value, and means only hard work, and worry, and humiliation has been passed on to succeeding classes and generations. To this unfortunate reaction my classes were no exception. Hence my experiment and this paper.

I tried my best to bring about a better attitude. I spared no efforts in trying to make tenth grade girls feel something of the fascination of the world of geometric thought. I not only did ample justice to the utilitarian values of the subject, but I also tried to make my pupils see that in comparison with its artistic and philosophic character, its utilitarian values become negligible. I tried to impress them by telling them of the age and dignity of geometry and its great contribution to civilization; how, long before the first textbook in this field was written, geometry had played a prominent part in the Valley of the Nile; how the textbook compiled in about 300 B.C. had survived until the present day; and how the greatest minds the world has ever known, had looked upon mathematics with reverence and enthusiasm. Many of these great minds, moreover, I tried to show them, were interested primarily in fields other than geometry and yet were fascinated by this science. I spoke, for example, of Plato, who made geometry the first requirement for entrance into his school of philosophy, and who saw in it a means of raising the mind from the things of sense to the invisible world and ultimately to the knowledge of the Idea of the Good or God; of Pythagoras, who by means mathematics was able to conceive for us the sublime idea of the Music of the Spheres; of Omar Khayyam, who had added lustre to Persian mathematics by his treatise

on Euclid; of Michel Angelo and Leonardo Da Vinci who had made to mathematics contributions only a little less than those of art and sculpture; of Dürer, who published the first treatise on the theory of curves, and of Napoleon, who said, "The advance and perfection of mathematics are closely joined to the prosperity of a nation."

Now, I told my students, *they* were to have an opportunity of learning something about this great subject of geometry. By careful study of its demonstrations and problems they would develop habits of logical thinking, of precise and succinct statements—they would learn to discriminate between the true and false. This all sounded very beautiful indeed, but learning two propositions a day (a necessity if one were to cover all the then required work each semester) and working as many original solutions and applications as possible, and then taking tests and examinations at stated times, made quite another story. Semesters came and went—and June counted up its A's, few—if any—B's slightly increased, a large number of C's and D's and, alas, too many F's. Then came September and oh, what a falling off in third year mathematics registration. After a few such experiences I felt in need of something, I did not know exactly what, but I thought my principal would know, and supply it—and she did. She was sympathetic, at least I think she meant to be. She brought my interview to a close with the most effective stimulus I have ever received. She said, "Well, don't worry about the situation, I am perfectly satisfied. I think you have done very well indeed. Mathematics is too hard for most girls—it requires a peculiar ability which very few of them have. It has little practical value for women. They spend time and energy with little lasting results. Even if they make their grade they forget in a few years, in a few months, indeed, almost everything they learned except perhaps the Pythagorean theorem, and they remember that only in name. Perhaps it's just as well that the large majority spend their time on cultural subjects."

I was stunned—little practical value—forgot all they learned—no cultural value—so that was how she felt—no doubt that was how the girls felt—everything was wrong. I was not primarily concerned as to whether my pupils would remember the demonstration of any particular proposition. I had expected some practical value in the form of transfer of training, and I had expected a great deal of cultural value. My tests showed

that not many could demonstrate theorems nor work originals—but I had no satisfactory way of testing transfer or cultural outcomes. However, one disconcerting fact was obvious—I had not succeeded in making the students like geometry. They were not interested enough in it to spend another year at it for its own sake regardless of high or low grades.

That it could be made interesting, of that I was sure, and I resolved then and there to leave nothing undone to make it so. I read everything I could find on outcomes and objectives in the teaching of geometry. I haunted the libraries for books and periodicals on the subject. I read history after history, gleaning those ideas which seemed to fit my purpose. I examined every modern textbook I could find for the best they had to offer, and in the meantime I tried out my ideas on individuals and small groups, on young folks and sometimes on grown-ups and usually I found the reaction favorable, so that when September of 1926 came I felt ready to subject my plan to trial. I was given the tenth year class of Derham Hall to do with as I pleased. One thing only was I to bear in mind; namely, that all responsibility for success or failure was to rest on me. But I was willing to assume the burden.

My program was to popularize geometry, that is, to bring about a favorable attitude of mind—to break down prejudice and dread—to build up interest and enthusiasm and a desire for further consideration of the subject. This was to be accomplished in an informal way through contact with nature. The panacea of all my *ills* was to be found in the most commonplace creatures of the organic and inorganic world. If I may be permitted to borrow the *use* of a quotation from Michael Pupin's *New Reformation* I shall say of myself, what he said of the electrical engineer:

"He keeps always in his mind the words of Holy Scripture: 'Seek not ye what ye shall eat, or what ye shall drink, neither be ye of doubtful mind.

" 'But rather seek ye the kingdom of God (Nature), and all these things shall be added unto you.' "

I sought not the practical or utilitarian—I sought only the cultural, that intangible something by means of which one sees in the ordinary things of creation the extraordinary workings of the Creator. With this objective once realized, interest in the

abstract beauties of geometry is bound to follow, and therein shall, "All these things be added to you."

Now for a number of years much of my interest outside of school work has been bound up in gardening. I have a flower garden—an old-fashioned one. I spend a large part of my leisure time in it. It has been the source of much inspiration to me and I meant to make it inspirational to my class.

After the customary formalities of the first class period, without any further ado about necessary books and supplies I told the class that we were going for a walk—that we were going out to see a new world—one which perhaps they had never seen before. This idea of a geometrical world was a "hard saying" to most of them. That houses, churches, and in fact, all buildings were geometrical they were willing to admit, but that there was aught of geometry or mathematics in trees and flowers, in the animals that played on the earth, the birds in the air, the sun and stars of the heavens, this was, to say the least, ridiculous.

On the way we talked about great men in the field of mathematics and especially those who had a large part in the beginning of its science. I mentioned Thales, Pythagoras, and Plato. I told them about Plato's attitude toward geometry—how he taught in a garden over whose gate he had written the first college entrance requirement, and with a well defined purpose in mind, I told them that, according to Plutarch, Plato had once said, "God is eternally geometrizing." Whereupon one saucy little imp turned up her pretty nose and said, "Oh, how perfectly stupid! I'm glad I'm not God." (It may be of interest to know that she was among the first to change her mind.)

On the way to the garden we noticed the trunks of the old oaks so tall and straight, the poplars less sturdy in their swaying loveliness, but straight as the oaks, and many other trees each having its distinctive beauty but possessing in common the power and seemingly the desire, to stretch straight up, to meet the rays of the noonday sun. Thus we came to learn the meaning of the term *perpendicular*. Then, looking through the trees and seeing numberless trunks, all stretching in the same direction we learned the meaning and beauty of parallelism—all lines perpendicular to a plane are parallel.

By this time we had reached the garden. The names of many of the plants were unknown to the members of the class. on learning the name of one, however, they could easily find more

of the same kind. They had recognized sameness of color and of form, but especially *form*—that geometric property which geometricians call *similarity*. From the idea of similarity there was only one step to *congruence* and numberless examples of the latter concept were at hand. This practically closed the hour for the first day. They returned to their class-room a little less skeptical of the existence of a geometric world.

The next day I questioned them on the previous day's expedition. Some had forgotten the new terms, some were hazy as to the ideas I had tried to teach. We therefore decided to go out for another walk, but to take notes this time. Our procedure was much the same as on the previous day, but it was not time lost. The girls were making more nature contacts—the geometric world was widening. The next concept which I tried to develop was that of *symmetry*. This idea of symmetry was wonderful. It revealed beauty which the girls had never noticed before. We found symmetry in trees, in twigs, in leaves and flowers. They learned to speak of *axis of symmetry*, which divided the leaf into two congruent planes, also of the *plane of symmetry* which usually divides the blossom into two equal parts. It was fun to find the plane of symmetry of the snap-dragon, as he opened his wide mouth, of the sweet-pea and the gladiola, of the lark-spur, of the Canterbury bell, and of the fascinating thimbles of some late foxgloves. We learned something too of another kind of symmetry—that found in the cosmos, the daisy, the calendula, and the Mourning Bride. We called this point symmetry and found that each flower had many axes, each of which was *bisected* by a *point* in the center of the flower.

This was the work of several days. In the meantime, because this geometric world had seemed too beautiful to leave to its fate in the autumn frosts, we gathered all kinds of leaves and blossoms and pressed them. We gathered the most beautiful specimens—those which showed the most perfect development of the *geometric* properties. For it was after all, as they were actually beginning to perceive, the *geometric* properties which made these beautiful things so beautiful.

Mindful of my own specimens carefully stored away for future use, I felt safe for the coming months. And with these newly found treasures which the class had gathered and lodged between the pages of their Montgomery Ward Catalogs I knew they were well provided for the work to come.

This work had taken about two weeks, and we now spent a day or two in organizing our notebooks. Each girl provided herself with a loose-leaf book, and divided it into five sections. The first was reserved for new terms; the second for terms requiring definition or illustration; the third, for geometric ideas; the fourth, for informal proofs or intuitive inferences; and the fifth, for historical notes or references. I might say here, that when the pupils became more interested in the abstract phase of geometry, they cultivated their texts and found that they could find therein much which they had written in their books. So sections one, two and four, having served their purpose, were immediately discontinued. The geometric idea section continued to be interesting throughout the year. In it could be found pictures from the brown sheet section of the newspaper, from the advertising section of magazines and from railroad folders, mounted nature speciments, original designs and copies of symmetric ornamental designs. In this section one could also find suggestive titles for themes and stories, for playlets and posters, all illustrating something which had been of interest to them and which they thought might be of interest to others.

Our next excursion was to our chapel, which is rich in all manner of symbolic ornaments. We studied them carefully and found that they were all geometric in design, and that their beauty was the result of their perfect symmetry. In the rose window which we examined rather minutely we found a marvelous specimen of point symmetry. It was of extremely simple design and the girls were unanimous in wanting to reproduce it, as well as some of the symbolic figures. This manifestation of enthusiasm was the reaction for which I was striving, and needless to say, I made the most of it.

Now, given the attitude toward, and the knowledge of, symmetric figures which these children had, added to the urge which had prompted their request, the teaching of a symmetric construction was a pleasant task. The pupils made several designs involving both kinds of symmetry and in so doing, not only mastered the *five fundamental constructions,* but discovered for themselves the three conditions of *congruency of triangles.*

By this time they were quite pleased with themselves—they had a fair opinion of their geometric ability, and their enthusiasm, plus a few well timed and carefully chosen stimuli, saw them through the First Book.

The middle of November found us ready for the Second Book. We were about to begin the study of circles. Before beginning, however, we got out our Montgomery Wards and I produced a beautifully illustrated seed catalog. (I do not think I should feel satisfied now if I had not referred to a seed catalog a few times in the course of a year's work in geometry.) Did circles figure in flowers? Yes, we found that a great number were *circular* in general form, and that some gave even more suggestion of circles. The daisy has a well defined circular center. The summer chrysanthemum revels in *concentric* circles of alternating colors. The calliopsis also abounds in concentric circles, but each blossom seldom gives us more than two, while certain species of phlox give us one tiny but well defined circular center in contrast with its outer edge.

But were circles confined to the flower world? Had the girls ever noticed them any place else in nature? These and similar questions seemed to open up a new line of observation. We are surrounded by one huge circle, the horizon—the outline of the sun in the sky is a circle—etc., etc., and many members of the class felt sure that if they had a little more time to think and observe, they could find many more instances of circles in nature.

Of this I assured them that I had not a doubt for the circle was one of the oldest symbols in the world. I told them that someone had once, beautifully said, that "the handwriting of the finger of God is the pageantry of the sky" and that according to a nebular hypothesis, the spiral was the first symbol to appear, and the circle, or its near neighbor, the ellipse, was the second. The next curve which appeared in this pageantry was one traced out by a visitor—the comet. The comet came into our system from a far distant realm generally called *infinity*, circled around our sun and left us again, usually, indeed, never to return. The path of the comet is often called a parabola.

Now sometime, I told them, if they continued studying mathematics, they would learn much about these curves. By means of the interpretation of these curves mathematicians and astronomers had discovered worlds of information about the universe, but for the present our task was to find out as much as we could about only one of this group, the circle. It was true that it seemed as if mathematicians had found out overything about it already, but, since it was a symbol of God's own handwriting perhaps it had a message for each one of us, in particular.

The girls never told me whether they had found His message or not, but they did tell me after that consideration, that a circle had a richer meaning for them—it was more than a commonplace curve made by a pencil fastened to a string, for it was nothing less than the second symbol in the pageantry of the sky; and secondly, that this consideration made the theory of its *chords* and *tangents* and *secants* and *intersections* far more interesting than they could otherwise have been. In fact, my little curly-headed lass said, "The tangents and secants seem like great long lines on which the circle could be rolled or slid right up to its place in the milky-way of the sky."

Now, toward the end of the study of the circle some authors introduced a few original problems in the construction of spirals. I seized on this opportunity, not because I thought it would help them to measure up to grade, nor because I thought it would be a stepping stone to the next book, but because I thought it would help "to achieve a life of rich significance." Could one or two days be better spent?

The pupils already knew the spiral as the first in order of time, of a group of curves, and so I freshened their memory by showing them pictures of the great Nebulas in Andromeda and Orion.

Now although, to look upon the picture of the Great Andromeda and the great Orion, was lovely and gave us a very good idea of their stupendous grandeur, nevertheless they were hundreds of thousands of light years distant from us and we with the naked eye could only catch a hazy glimpse of them. But perhaps as a sort of compensation for their distance from us, the symbol of the spiral had been brought down to earth, and a little observation would find it for us in both the plant and the animal world.

Since a visit to the garden was impossible, our specimens and the seed catalog were again brought into use. We were not long in finding that the arrangement of leaves around the stalk in many plants was that of a spiral, that the sections of the cones of the pine tree also followed the same arrangement, and that the seeds in the heart of a sunflower are spirally arranged; a specimen of Queen Ann's lace presented a perfect and delicate spiral, and had I been as fortunate two years ago as I am today I could have shown them a living specimen of this curve in the leaf of a begonia presenting the most perfect spiral curve that one could imagine. Simple as it is to find illustrations of these

curves in the plant world perhaps it is even a simpler matter to find them in shells. In our case it was especially easy, for just two doors from our classroom we had a whole collection of shells illustrating spirals of various kinds from the enchanting Chambered Nautilus, revealing both interior and exterior views, to the common snail. Here we have come face to face with indescribable beauty, which continued to give inspiration to all who would look upon it with seeing eyes. And what was the underlying principle of all this beauty? Purely and simply a mathematical law which governed their proportional parts.

The immediate reaction was surprise and admiration, followed by wonder and conjecture—the remote reaction beautifully expressed in short themes and sketches which I assigned them later gave evidence that the time, to say the least, had been well spent.

In the plan for the year's work I had hoped to be ready for Book III by the beginning of the Second Semester. Nor was I disappointed, for we were more than ready—we had a day or two to spare.

In order that this paper be not entirely out of proportion to the total space allotted for it I shall be obliged to cover the remainder of the course with all possible brevity. I shall touch upon only those parts which have direct bearing on the main purpose of its development.

In elaborating the properties of *similar triangles* we had recourse again to the specimens which we had previously used to bring out intuitively, the idea of similarity, as a whole. The trillium was especially useful for this purpose, for by measuring the lines which join the corresponding points in two trilliums, assumed to be similar, it is quite easy to show that the ratios formed approximate equalities. Likewise the equality of corresponding angles can be shown by direct measurement. Leaves of the violet plant can also be used to advantage for the same purpose, for it can be readily seen that the lines joining the outstanding points of these leaves form a triangle. For the other similar polygons, beautiful illustrations can be found in the leaves of poplar trees, maples, cotton woods, horse chestnuts, and ivy vines.

For the work on *regular polygons,* exquisite flowers abound as examples for figures of almost any number of sides. Spring violets, apple-blossoms, wild roses, four o'clocks, forget-me-nots,

morning glories, and nasturtiums can all be used to bring out the properties of pentagons.

Lilies, narcissus, the jonquil and the asphodel are marvellous illustrations of hexagons. But if I failed to emphasize the regular hexagonal properties of snow crystal, I should feel that I had missed one of the best opportunities of helping my pupils to see, in the commonplace creatures of earth, the marvellous work of an omnipotent hand.

Examples of heptagons are not so plentiful. As I recall now I can name only one flower, *love-in-a-mist*, that beautiful garden annual, so frail and exquisite in its pale blue and yet so easily grown, which though it has more than seven petals yet it almost invariably sends out seven tendrils which enfold its blossom in a misty green. Abundant and beautiful examples of octagons are seen in the cosmos. And the daisy in some one of its many species will furnish examples of polygons of almost any desired number of sides.

And now there remains but one topic to be developed, namely, the result. As I recall my quotation, the use of which I had borrowed from the *New Reformation*, it ran as follows:

> "Seek ye first the kingdom of God
> And therunto shall all these things be added."

I fear that you will be inclined to think me guilty of an inconsistency because in talking of results I am going to begin with "those which were to be added." My reasons for this are threefold: (1) I am anticipating the question; (2) we have accepted means of measuring such results, and (3) it is easier to talk about these concrete outcomes than it is to talk about these other less tangible, though none the less real values, which were the main objectives in this experiment.

By the end of May we had covered all the propositions listed as minimum essentials by the National Council in the report of 1923, and, in addition to them the pupils were familiar with a number of ideas ordinarily listed as solid geometry theorems. We measured these results by means of the Schorling Sanford Achievement Test.

In the class of 1926–27, 26 out of 32 reached the median and in the class of 1927–28, 24 out of 28 reached it.

Of the realization of this other objective, the objective of primary importance in this experiment I shall have to speak with

less concrete evidence, though I may add, none the less convincing ones. In spite of the reiterated opinion on the part of educators in the field of mathematics that of all values the cultural or esthetic values far outweigh all others, no adequate norm has ever been prepared for their measurement. Hence my conclusions are based largely on observation, casual statements, and significant expressions of appreciation.

In *The Humanizing of Knowledge*, by James Harvey Robinson, we read a statement embodying the idea that an effective course in science ought to foster such a lively understanding of the workings of nature that the fascination of discovering ever new wonders *will endure through life* and mitigate sorrow, boredom and disappointment. Again, in an article read at the National Education Association in the Department of Superintendence in Dallas, Texas, the following significant statement was made: "The greatest joy a man can have is when he is able to think God's thought after Him and rejoice with exceeding joy." Now because of the absence of a norm for measuring the cultural objectives I used as a standard in evaluating my findings the above mentioned quotations, together with that familiar thought from Tennyson,

> "Little Flower—but if I could understand
> What you are, root and all, and all in all,
> I should know what God and man is."

On learning of the pleasure which the great majority of my students had experienced in the actual class hour, and of the wish, and request, on the part of a considerable number that they be allowed to continue work in mathematics for at least another year, I concluded that I had developed *real live interest*. When I consider the persistence with which my students frequent my garden, always alert for any new bit of information which I might be able to give them or which they might be able to find, I feel that I have developed the desire "*of discovering ever new wonders.*" And when I see the exprssion of joy on the face of a child as she brings me or tells me about a new specimen of a plant or design in which she has found that which to her is a new meaning or interpretation, I feel that I have helped her in a small way at least, in "thinking God's thought after Him and in rejoicing with exceeding joy," and that through this discovery she has learned a little more of what "God and man is."

A COMPARISON OF THE MATHEMATICS IN THE SECONDARY SCHOOLS OF GERMANY AND ENGLAND AND THE TRAINING OF TEACHERS OF SECONDARY MATHEMATICS IN THESE COUNTRIES WITH THE UNITED STATES

BY JAMES H. ZANT

State Teachers College, Durant, Okla.

MATHEMATICS IN THE GERMAN SECONDARY SCHOOLS

I. Organization.
   1. As parallel units.
   2. Extends over the whole system.
   3. Essentially the same in all courses.
   4. Course of study.

II. Content.
   1. Arithmetic.
   2. Geometry.
   3. Algebra.
   4. Trigonometry.
   5. The calculus.
   6. Historical and philosophical survey.
   7. Functionality.

III. Standards.
   1. Range of subject matter.
   2. Mastery of subject matter.
   3. Number retained in school.

I. *Organization*

From a reading of the "Richtlinien"[1] one is likely to get the impression that the mathematics of the German schools is of the "correlated" type. One of the distinctive features of its organization is that not all of algebra or all of geometry is taught in one year or two consecutive years as has so long been characteristic of our American schools. The statement by Beatley in the MATHEMATICS TEACHER,[2] "The intention is rather to maintain

[1] Translated "Suggestions." Issued by the state for the teachers in German Schools. The course is made up by the teachers from these suggestions.

[2] Beatly, Ralph, Book Review, THE MATHEMATICS TEACHER, Vol. 20, p. 412.

the continental traditions of teaching algebra and geometry in continuous parallel courses over a period of several years, emphasising whenever desirable the numerous pertinent applications of one subject to the other, but at the same time keeping the two essentially distinct and intact, and so giving an appreciation of each by itself as well as an appreciation of each in relation to the other," is probably a much better expression of its organization. Just how far these parallel courses are carried is not indicated. Probably the higher parts of algebra are carried as a parallel to the courses in analytical geometry and the calculus. Certainly there are many places in each of the subjects which would be much clearer if handled at the same time or near the same time as other parts of the related courses.

2. Another significant characteristic of German secondary mathematics is that it extends over the whole school system. Apparently it is taught from the beginning of the secondary school through to the last year. This leads to the study of a rather wide range of topics, which include a number of units in projective and analytical geometry and the calculus as well as the more advanced topics of algebra and trigonometry.

3. There is a striking likeness in the suggestions for all the different types of schools. The differences in any given grade or year are all minor ones, many times a topic included in one sort of school (U II,[3] for instance) may appear not in this particular year in the other type of schools but will appear in most of them (in O II or possibly O III).

4. Probably the most significant thing about the German secondary system is the method in which the course of study for a given subject is evolved. The "Richtlinien" is issued by most of the states. The course of study for this subject must be justified before the whole faculty before it is adopted. Not all of the suggestions for the course then may be used; only those which the teacher feels will best give the desired ends under local conditions.

[3] The German secondary schools cover a period of nine years, which are designated, VI, V, IV, U III, O III, U II, O II, U I and O I, O I being the last year. The U and O stand for the words "Unter" and "Ober" translated "Lower" and "Higher."

II. *Content*

1. *Arithmetic.*—The formal teaching of arithmetic is confined largely to the first two years of the secondary school and it is built on the lower school work. The work begins with fractions and carries on through compound numbers and percents to applications and a seemingly rather simple business arithmetic. Short cuts and some graphical representation are also included. A warning that the informational arithmetic must not smother the real arithmetic is given. There is also a provision which states that calculation both oral and written shall continue throughout all the other courses in mathematics.

2. *Geometry.*—The geometry of the German schools covers a wide range from simple intuitive geometry and drawing to analytic and projective geometry. The intuitive geometry is begun early in the fifth or sixth year and continued until probably the ninth year along with the other phases of mathematics taught. This is intended to give the pupil a good understanding of the names and notions of geometry before he takes up demonstration. There is evidently not a distinct line of demarcation between plane and solid geometry. Analytical and projective geometry are apparently taught largely in connection with other branches of mathematics, for example, plotting curves is listed as a part or in connection with the study of their equations in algebra.

3. *Algebra.*—Algebra is begun early in the sixth or seventh year and continued in some form throughout the whole period. It seems to be begun in earnest in the eighth or ninth year and carried as a parallel subject with geometry for the two years following, finishing quadratics in the ninth year and going on to the higher parts the following year. The amount of pure algebra decreases then to make way for the work in analytics, projective geometry and the calculus.

4. *Trigonometry.*—Trigonometry is taught mostly in O II and contains what is usually considered a complete course in American schools, that is, through spherical trigonometry, plus applications to astronomy and mathematical geography.

5. *The Calculus.*—The calculus offered is limited to simple differentiations and integrations with applications to rational and trigonometric functions, for calculation of their maximum and minimum roots. Also the calculation of area and content

by integration using simple curves, circle, parabola, sphere, paraboloid and the like.

6. *Historical and Philosophical Survey of the Course.*—The review that is included at the end of the school course is rather striking and gives opportunity to see the course in mathematics as a whole. Along with this the history and philosophy of the subject are introduced. Since the German teachers are well qualified by training to do this, it should be one of the important contributions to the pupil's knowledge.

Another thing that might be added under the head of content is the definite effort made to get real application in all of the school mathematics. This should aid greatly in the pupils understanding and appreciation of the subject.

7. *Functionality.*—The idea of functionality is also given considerable attention in the teaching of German mathematics. During the first three years of the secondary school it is developed through the study of first and second degree equations. Near the end of the course it is more strongly and carefully emphasized by the methods of the calculus which gives the best type of illustration.

III. *Standards*

1. *Range of Subject Matter.*—The standards in the range of subject matter taught is comparatively high in the German schools. The depth in each separate part of mathematics is far greater than many of us thought it possible to carry high school students. One or two things must be kept in mind in regard to the scope of the curriculum in mathematics. The course includes two years of the traditional college work, and not all the subject matter is compulsory; sometimes one thing is omitted and sometimes another.

2. *Mastery of Subject Matter.*—The standard in the mastery of this subject is apparently very high. The methods of teaching and the very subject matter itself indicate that this is the case. No student could succeed in the mathematical units offered during the last years of the German secondary school without a rather superior mastery of the more elementary courses.

3. *Number Retained in School.* From another view point the German secondary school does not stand out so well. The number of pupils finishing as compared with those who enter is very small. Some estimates have it as low as 6 to 100. A very large

percentage of pupils leave on completing the first six years of the secondary course, that is, at the age of sixteen. This is probably due to a large extent to the rather restrictive standards required. On the other hand higher standards may be expected for several reasons:

(1) Schools are selective.
(2) Those who finally graduate are selected from these.
(3) More years are spent in the secondary school.
(4) The school year is longer (250 days).
(5) Home work is required (at least 2 hours, sometimes 6).
(6) One must be educated to succeed.
(7) Schools have well trained teachers.
(8) A teacher's position has dignity.
(9) There is more drill.

Mathematics in the English Secondary Schools

Outline

I. Organization.
   1. Course of study.
   2. Not controlled by the examinations.
   3. Not necessary to include all of the recommended course.
   4. May include something not set on the examinations.
   5. How do they get a national system?

II. Content.
   1. Minimum course for the first examination.
   2. Course for the second examination.

III. Standards.
   1. Range of subject matter.
   2. Mastery of subject matter.
   3. Number retained in school.

I. *Organization*

1. The course of study in the English secondary schools is even less prescribed than in the elementary schools. Nothing is definitely prescribed, even a Handbook of Suggestions is not issued. Each school under the guidance of its principal makes its own course of study. This course must be approved by the school inspector and approval is usually obtained if the principal is confident of his course.

2. All pupils in grant-aided schools take the First Examina-

tion at about the age of sixteen. It is designed to test a sound general education. It also serves as an admission to the universities. A mere "pass," however, even with a high mark, is not sufficient for this purpose. The candidate must take a more narrowly specified list of subjects and make a much better showing on each in order to obtain "credit" standard. This list varies with the different universities but there is much resemblance. The pupils are then granted the school certificate with or without the matriculation privilege. These are often used in showing their degree of attainment when applying for positions. Those with the matriculation privilege are of course of most value and many employers will not fill their positions with any other kind of boys.

The second examination was set up in 1917 in order to encourage some of the pupils to remain in school two years longer and go further along special lines of work. It is designed "to test the results of specialized study of a coordinated group of subjects combined with more general knowledge of subjects outside of this group."[4] This examination also serves two purposes, a sufficiently good pass in it may also be accepted as excusing the student from the first year of the university course for a degree.

Under such conditions one would expect to find these examinations controling to a large extent the curricula of the schools. Many English school men have said that their effect is not altogether good. They point out the fact that few go to the university but all must take the examinations.

These evils are offset somewhat however by several factors that are in operation. Among these may be stated:

(1) The Secondary School Examinations Council set up by the Board of Education in 1917, composed of ten representatives from the examining bodies, six from the Teachers' Registration Council and five from the Association of local authorities. Its function is to make the examinations acceptable to as many schools as possible; they have had some degree of success.

(2) A secondary teacher, or one who has been a secondary teacher, sometimes not very recently however, is appointed to help set the questions. There still seems to be consider-

[4] Kandel, I. L., *Education in Great Britain and Ireland*, U. S. Bureau of Education, Bulletin No. 19, 1919, p. 32.

able criticism on this score; some say that these ex-secondary teachers are so far removed from the class room that they are completely out of touch with it.

(3) The teachers often request, and have their requests granted, that certain phases of a subject be represented in the examination. In this way the calculus that is taught in England is usually recognized by the examining boards.

(4) The group rather than the individual subject is the unit in determining success or failure. This principle if carried out gives the individual schools quite a leeway in developing their subject matter in a course like mathematics and still be fairly sure of success in the examinations.

3. Another feature of the organization of the subject matter in mathematics is that no school has to include all of the topics listed by the Joint Matriculation Board or the other examining boards. One isn't required to answer all the questions on any examination and there is some choice among the examinations one may take.

4. As stated above the courses of study often include topics that are not listed by the examining boards. These schools may request that questions be set on these topics or they may simply teach them because they are considered valuable for the pupils.

5. All these things lead one to wonder how the English have attained a national well organized course of study. There are several controls however which give to the English schools a fairly definite course of study for mathematics as well as the other subjects. They are:

(1) The national ideal; all teachers or makers of courses of study think alike along certain lines.

(2) Preparation of teachers; all are trained the same way.

(3) Purpose of the school; the extent of mathematics that can be taught is limited by the age and development of the child.

(4) The inspector; his function is to make suggestions, in all but the strongest schools his advice is usually taken in the end.

(5) Training of the teacher in service tends to give all the same ideal.

(6) Professional organizations of teachers lead the same way.

It might be noted that the reason for teaching mathematics or any other subject in the English schools is to assist in attaining their national ideal of individual development. With this in mind it comes to matter little just what courses are taught or what particular subject matter is included in each course. They are not primarily trying to teach mathematics but to teach children. What they teach is not based on the theory of what the children ought to be able to learn, but on what children have actually been able to accomplish.

II. *Content*

1. The content of mathematics in the English secondary schools will be discussed first from the stand point of the minimum course, which is usually completed at the age of 16 and followed by the First Examination. The passing of this examination as stated above may entitle the student to a School Certificate or if the paper is of the required quality it may serve as the matriculation examination for the university. The Joint Matriculation Board, as well as the universities who set their own examinations, issues a list of suggestions for this examination. The subjects included are arithmetic, algebra, geometry, and trigonometry. The arithmetic, of course, is based on the elementary school work and begins with common and decimal fractions, takes up percentage and business arithmetic, mensuration, plane and solid, square root, and simple graphs. This makes up a fairly definite course and with proper placement and teaching may become a very valuable course for the pupil. The only change in the syllabus is the introduction of logarithmns for the whole group.

The algebra extends through quadratics, in addition questions are set on graphs of linear and quadratic functions, simple theory of quadratics, logarithmns, simple surds, simple ratio, proportion and variation. This is equivalent to about one and one half years of algebra as we teach it. There has been little change in the content or method of teaching or the results in recent years. "Compared with the size of the tool, the pupils have made, as yet, but little use of it."[5]

[5] Carson, G. St. L., The National Council for Teachers of Mathematics. Four Year Book, Bureau of Publications, Teachers College, Columbia University, New York, p. 24.

Geometry for the School Certificate covers the equivalent of the first six books of Euclid. This is divided into Practical and Theoretical Geometry, the Practical being made up of constructions and easy extentions from them, these constructions are listed. The Theoretical Geometry is made up of theorems and easy deductions from them. These theorems are listed and consist of the ordinary theorems of geometry. There has been practically no change in recent years in the content of geometry but much in the methods of teaching, much more is being accomplished with the pupils at present than fifteen years ago. It is significant that "there has been no change in the maximum content of the standard syllabus and until recently no suggestions of change."[6] Teachers have used their time to work out a proper treatment of what has been offered but, "signs are not wanting that the next general movement may be in the direction of an enlarged syllabus or range of study, on the ground that the present content is too small to give an adequate back ground of geometrical experience."[7]

The trigonometry for the first examination is evidently very limited consisting of the functions of an angle of less than 90° and simple solution of right triangles. Its popularity is probably growing and it is being taught to more pupils, both boys and girls. In practice it has been found to be the step by which many boys and girls have found "an interest in and a power over their mathematical work."[8]

2. The second examinations given two years later at the age of eighteen have been described above. Mathematics seems to be one of the most popular subjects chosen during this time. It is essentially a period of specialization and with a required standard will exempt one from the first year at the university. Since the provision for this examination was only made in 1927 the mathematics syllabi for this work are not very well developed. The examining boards have suggested certain extension of the existing courses in algebra, geometry etc., but there has not been time to work them out definitely. There are also new subjects added which include analytic geometry, the calculus and a combination course of mathematics and mechanics. The work done so far represents more or less what able pupils can achieve

[6] Carson, G. St. L., *op. cit.*, p. 25.
[7] Carson, G. St. L., *op. cit.*, p. 25
[8] Carson, G. St. L., *op. cit.*, p. 26.

in two years when studying mathematics along with one or two other principal subjects.

A statement regarding the calculus may well be added. It seems to be in line for the next development. Some schools are already teaching it with notable success. It does in fact appear on the first examination in the topic "Advanced Mathematics," taken by a very few as yet, as a substitution for or alternative to the bulk of the traditional advanced algebra, abstract trigonometry and analytic geometry. The principle stated by Mr. Carson, "it is in calculus if at all, that the formal algebra now taught will find its proper aim,"[9] will probably mean much to the development of the teaching of algebra as well as the calculus in the English schools.

III. *Standards*

The discussion of the standards of mathematics in the English secondary schools would probably be about the same as that made in the German schools, with the exception that the First Examination is not supposed to cover as wide a range of subject matter as is covered in the German schools. The remainder will not be repeated here.

Comparison of the English and German Secondary Mathematics with that of the United States

As to Organization

1. *Differences.*—

(1) In the United States the courses in mathematics are still taught as separate courses with little or no connection with each other, that is algebra only is taught for a year and then geometry. In Germany they are taught as parallel courses with two or three days of the week devoted to algebra and the others to geometry. The same is true generally throughout the school course in mathematics.

(2) In the United States the course of study is largely prescribed by the state or other organizations. There is very little liberty on the part of the teacher as to what he teaches. In Germany the state issues suggestions but the teachers make the course of study from these and may leave out or add topics as they think advisable.

[9] Carson, *op. cit.*, p. 27.

(3) Much more attention is paid in Germany to the applications and correlation of mathematics. Each department in the school definitely attempts to aid the other departments and bits of the history of mathematics are put in literature lessons etc. Altogether too little of this is done in American schools.

(4) The mathematics of the secondary American school is probably still more influenced by outside examinations than those of Germany. This is true of the schools in the eastern part of the United States but not so true in the south and west probably, where the students enter the colleges on high school certificates. These schools are, however, greatly influenced through the text books in use, which are usually prepared for schools where the examinations are given.

In England the same difference could be found with a few minor variations. No suggestions for courses of study are offered at all in England by the state, the teachers make their own courses but must have the approval of the inspector. The Matriculation examinations probably have a more profound effect than in Germany but are not wholly without their good results.

2. *Likenesses.*—There seems to be few things about the organization of the subject matter of mathematics in these two countries that are like those in the United States. German schools have mathematics throughout the whole course and this is also true in the United States but it is almost wholly elective after the second year and comparatively few take it, this is possibly true to some extent also in the English schools. The newer method of organization of the mathematics of the junior high school in America is somewhat like the organization of mathematics of the German system.

As to Content

1. *Differences.*—The difference in content is largely of degree, the German schools have their pupils two or three years longer, have longer days, better teachers etc. as discussed above. As a result the work offered in mathematics goes much further than we do. These things have been discussed in the first section of this paper and include:

(1) Considerable stress on functionality.
(2) Strict subject matter, higher algebra, analytic and projective geometry, abstract trigonometry and the calculus.

(3) A philosophical and historical treatment of the subject.
(4) A review at the end of the course which covers everything the pupil has had.

The grades in which topics are introduced vary somewhat from ours. Ordinarily any topic is introduced in German schools earlier than in the United States.

Intuitive geometry is introduced in the fifth and sixth year and algebra as early as the seventh year. This is probably due to some extent to the longer school year. By the end of the ninth year the German pupil is at least a year ahead in mathematics of the American pupil in the same grade.

In England the mathematics course extends about as far as in the German schools. This is true for those who take the Second Examination but not for those who take the First Examination. Perhaps not so much stress is put on the calculus. For the School Certificate which is granted upon the successful completion of the First Examination the course is apparently about the same as that in the United States.

2. *Likenesses.*—The subject matter of secondary mathematics is about the same in the three countries as far as it goes. First year algebra and geometry are the same in the three types of schools. The same is true of the other subjects that are taught in the United States schools.

As to Standards

The standards in both the range of subject matter and the mastery of this subject matter are much higher in both Germany and England than in the United States. The reasons for this have been discussed above.

From the standpoint of the numbers of pupils who complete the courses, the United States probably stands much higher. More attention is paid to the individual pupil and his progress. This is especially true in a system of schools entirely supported by the state. In practically all the English schools at least a part of the students pay fees. Our pupils may not learn as much about mathematics or may not learn it as thoroughly but more of them complete the course.

The Training of Secondary Mathematics Teachers in Germany, England and the United States

Germany

1. *Requirements.*—(Primary education 3 years; secondary education 9 years.) The German prospective teacher is required to spend four years in the university where he specializes in subject matter. This is followed by a state examination consisting of religion, the German language, philosophy of education, two majors and one minor. Upon the successful completion of the examination he is required to spend two years under the direct guidance of some approved secondary school, during which time he studies theoretical education and is gradually introduced into the art of teaching. Following this the candidate takes a professional examination, written and oral. When favorable report has been made on this work "his name is put on the list of teachers eligible for appointment in the higher schools."[10] This is usually completed at about the age of 24 or 26 and at the present he rarely gets permanent appointment before the age of 30. "For first grade rank in pure mathematics the candidate must show that he is thoroughly aquainted with the principles of higher geometry, arithmetic, algebra, higher analysis, and analytic mechanics and can solve fairly difficult problems without assistance."[11]

2. *Tendencies.*—The system has been criticized severely, some have said that it is not a professional training but an apprenticeship, they merely learn to imitate and teach the subject matter. This is a way of saying that there is too much practice and not enough professional study. It is being urged that the universities assume the task of the theory and psychology courses in addition to the academic preparation. It has been proposed also that the candidate spend the first year of the "Seminare" in the schools getting some experience and then spend the next year in a group, with study and lectures on the theory of education. Saxony has made such a reform with some variations.

[10] *Reorganization of Mathematics in Secondary Education*, A Report of the National Committee on Mathematical Requirements, p. 438.

[11] *Ibid.*

England

1. *Requirements.*—(Primary education 6 years; secondary education about seven years). The main requirement for the English secondary teacher is an honors degree taking four years. Included in this is usually found twelve weeks of practice teaching of which two thirds must be in the secondary school, and theory work which covers, principles of education, educational psychology, hygiene, history of education, and methods of teaching the particular major subject. The amount of mathematics varies considerably due to the variation in the English secondary school.

2. *Tendencies.*—While professionalized training is recognized in the elementary school, it is not in the secondary school. The situation is improving however through the encouragement of the government, in the way of special grants for those who wish to teach and will remain a year longer for special training.

United States

1. *Requirements.*—(Primary education 6 or 8 years, secondary education 6 or 4 years.) Due to the youthfulness of our schools and many other factors, one may find people attempting to teach secondary mathematics who have had nothing beyond the high school requirement of a year of algebra and a year of geometry. Fortunately this condition exists in very few schools and is rapidly disappearing. The conditions in the North Central Association are probably near the conditions of our approved schools everywhere. Its requirement is that the teacher must have at least 12 semester hours in college mathematics before he can teach it in a secondary school. It is almost necessary for a person to have a B.A. degree before he can teach permanently in the high school.

2. *Tendencies.*—It is interesting to note some of the figures of the North Central Association for 1920 which may be considered tendencies for all our schools, especially those not the approved lists. 65.2 percent held the B.A. degree with 15 hours of education, 11.9 percent held in addition advanced degrees, more than half have had some graduate courses, 89.3 percent have had some professional training, 37 percent were trained only in part in the subject they taught, two thirds were teaching only one subject and one fourth more teach an additional subject.

Compared with the European schools these tendencies are not particularly high but they represent a step forward.

We hope that Professor Reeve is right when he says, "we ought to be able presently to require the calculus of all prospective mathematics teachers in the secondary schools." [12]

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[12] Reeve, W. D., The National Council for Teachers of Mathematics, Fourth Year Book, p. 142.

MATHEMATICAL OBJECTIVES

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The practical man, who frequently finds time between business and golf to lament the sins of the schools, is likely to insist that education be directed toward a definite goal. A considerable number of the students, on the other hand, still exemplify the refrain of the old song "I don't know where I'm going, but I'm on the way." Our pedagogical friends who, in the emergency, have accepted the weighty but voluntary task of rebuilding the curriculm, have adopted a catch-word made popular by the war. Before any course may be considered for the new curriculum, the expert must first ascertain its *objectives*—although his vague ideas of the objectives of education itself may defy formulation. "What are the objectives of your course in trigonometry?" one of these zealots will demand, in a manner that implies that the quaking victim is expected to "stand and deliver." On such occasions I am reminded of a colloquy that once occurred at a district school meeting when the school board was censured for squandering $32 of the sovereign taxpayers' money for a set of geometrical models, which, except for a physiological "chart," comprised the entire scholastic equipment of the school. "What do they use them *fur?*" demanded an irate father of a numerous progeny, none of whom it must be confessed ever profited by the novel luxury. "To teach the children mathematics" was the devastating reply of the director.

What are the objectives of the courses in arithmetic, algebra, geometry and calculus? To teach the children mathematics—what, are you answered Mr. Critic? Alas no, for whatever objective may be proposed, that of teaching subject matter seems to be obsolete. We have gone far—perhaps forward, perhaps backward, possibly along the arc of a circle—since Chief Justice Taft, in accepting the Kent professorship of law at his Alma Mater, announced in effect that it should be his purpose to acquaint the youth of Yale with the Constitution of the United States. Even our efficient school authorities, in the occasional

inventories of their educational wares—with a view to reducing the overhead—look with misgivings upon any commodity however staple that fails to record the standard turnover. They too would examine the objectives of mathematical instruction.

Now a mere enumeration of objectives is futile unless the objectives be worthy of attainment. I propose accordingly the following line of defense on behalf of the mathematician. Trigonometry, college algebra, analytics, and calculus are required subjects in most engineering schools, since they are indispensable equipment of a trained engineer, be he civil, electrical, mechanical or aeronautical. It would be presumptuous for a comparative layman to attempt a vindication of the colleges of engineering—that task belongs rather to the engineering profession.

Again, the same subjects are normally required or strongly recommended for major students in physics and chemistry while some of them are suggested for biologists and geologists. Physics, chemistry and biology in turn are required for entrance to the best medical schools. These several branches of natural science have been gaining ground during the past fifty years until they are now universally recognized as suitable studies in a program of liberal education—except possibly at Oxford and among the followers of Matthew Arnold. Let those who question them apply to the scientists. The medical schools alluded to are of course the schools of scientific or orthodox medicine, which have been the object of attack from the allied ranks of a multitude of dissenting cults. Whether a public university should take sides in favor of one to the exclusion of all the others is a problem outside the realm of mathematics—let those concerned consult the doctors of medicine.

Plato made geometry an entrance requirement to his philosophical academy. For argued Plato, "Geometry will create the mind of philosophy" since "the knowledge at which geometry aims is of the eternal and not of the perishing and transient." Pythagoras likewise and his school believed that geometry and number held the key to the riddle of the universe. But these men lived centuries ago and their ideas may need revision in the modern world. Much of the domain of ancient philosophy has been usurped by the sciences, including mathematics; witness the philosophic implications of the work of Einstein, Eddington, Weyl, Whitehead, Russell and a multitude of others. The skeptic who seeks an apology for such philosophy as remains

under the parent name is respectfully referred to the disciples of Plato and Kant on the university faculties.

Mathematics is a kind of language—a "divine shorthand" as one enthusiast expresses it—the most precise and abridged yet evolved, and truly international in scope. A recent book on pedagogy goes so far as to treat mathematics and grammar in the same chapter because of the common elements in the two sciences. Now language has been regarded as a desirable accomplishment of scholars since the days of the Vedas, before Homer sang or Demosthenes thundered. That this honored tradition has lost some of its former momentum however, is apparent from the decadence of the classics in recent years. Nevertheless, I am confident that the departments of language can offer at least some rhetorical reasons for the continuation of linguistic courses.

Occasionally a mathematical memoir is classed as news. Not many weeks have passed since Einstein's latest contribution to mathematical physics appeared in certain newspapers. At rare intervals, an article from the pen of a less notable mathematician is to be found in the Sunday supplements. Whether it is the function of a university thus to assist in the creation of news or merely to train students in the art of writing it, is a point that the schools of journalism might wish to debate.

Mathematicians agree that mathematics possesses qualities of beauty, analogous to those of poetry, music, sculpture and other forms of art. Professor Birkhoff of Harvard gave a course at this year's summer session at Columbia University on the mathematical elements of art. The trinity of the good, the true and the beautiful have enjoyed a sanction as general as it is ancient. But the justification of esthetics—if justification be demanded—is rather the province of the faculties of fine arts.

Mathematics furnishes the most accurate and adequate view of infinity to be found in any subject. In many religions, various infinite attributes are ascribed to the Deity, so that mathematics might throw some light on the nature of divinity. Indeed Plato asserted that "God continually geometrizes." But Professor Barnes, Attorney Darrow and Sinclair Lewis have challenged God. While modern youth, according to allegation, is godless—a natural consequence one infers from walking in the counsel of the ungodly. Whether, in a university supported by taxation, in a country where the separation of church and state is a cardinal

principle, there is a place for the teaching of religion is perhaps a question—but let the theologians, not the mathematicians, make answer.

Mathematics is fast becoming essential to the study of economics and the calculations of modern business. Life insurance is doubtless the most scientific as it is one of the largest and most important of business enterprises—and actuarial science is merely the application of mathematical principles to the statistical problem of life and death. He would be a rash man indeed who would question the propriety of erecting on every college campus suitable shrines, dedicated to the worship of the Almighty Dollar. Even the clergy applaud—at least the Rotarian branch of the clergy which is fond of asserting that Moses is the world's greatest real estate promoter and Jesus Christ the world's premier salesman. Need a mere mathematician lift his feeble voice to swell the mighty hallelujah chorus?

The foregoing will peradventure appear to the objective trailer as a naïve response, blinded as he is by the naïveté of his inquiry. Let us resort to the parable of the ancient sage who was accustomed to "explain" to the simple-minded questioners that the earth rested on the back of a huge elephant. The more sophisticated searchers, who were curious about the support of the elephant, were silenced by the assurance that the elephant stood on an immense rock.

We have now shifted the burden of justifying mathematics upon the ample shoulders of the elephant herd of the natural sciences, engineering, economics, language, philosophy, esthetics and religion. What do the elephants stand on? Why, the solid rock of human needs. And the rock? Ah, I was expecting that. Then, abandoning parables, pachyderms and pedestals, let it be perennially and eternally proclaimed that the study of mathematics fosters careful, accurate, sustained thinking, stimulating the while thinking itself. It strengthens the reason, develops the power of generalization, cultivates the imagination, and brings one face to face with chaste but naked truth. Was it Spinoza who said in substance that if mathematics—unlike history and politics—had not been independent of personal interest, the world should never have known truth?

In short I hold with the musing poet:

"Flower in the crannied wall,
I pluck you out of the crannies,
I hold you here, root and all, in my hand,
Little flower—but *if* I could understand
What you are, root and all, and all in all,
I should know what God and man is."

And this is as valid a defense of mathematics as of botany. Of no other subject can it be affirmed so completely as of mathematics

"Nothing useless is or low,
Each thing in its place is best
And what seems but idle show
Strengthens and supports the rest."

NOTICE TO OUR SUBSCRIBERS

We are planning to publish as a supplement to the January or February issue of THE MATHEMATICS TEACHER without cost to the paid-up members of the National Council of Teachers of Mathematics two important publications.

1. The long planned for "Register of Members" or "Who's Who in The Teaching of Mathematics." Those members of the Council whose dues of $2 were paid up at the time of going to press will all receive copies of the register.

2. A complete index not only of titles, but by authors of THE MATHEMATICS TEACHER from 1921 to 1929, inclusive, together with a list of the book reviews for the same period. Former subscribers may secure this index for actual cost plus the cost of mailing after the index is published. This ought to be of especial interest to libraries all over the country.

SUGGESTIONS FOR A FIRST LESSON IN ALGEBRA

By H. F. MUNCH

University of North Carolina

The following suggestions are designed to give the teacher an appropriate introduction to the subject of algebra. It probably should not take more than 25 to 30 minutes of the first day's work to present the material below. The objective of this part of the first lesson should be to develop in the pupils proper attitudes toward algebra and toward the teacher. Many pupils, when they come to the algebra class the first day, have built up a defense attitude or an attitude of dislike if not hatred for the subject. Such a lesson, if given by the teacher in a frank, open, friendly manner, should tend to change this attitude to one of tolerance if not to one of interest in and like for it. If this can be accomplished the first day, surely the algebra period will be well spent.

The work which we are to do in this class during the coming year may be compared to taking a sight-seeing trip. You know there are people who get together parties of tourists and take tours through the Holy Land, the mountains of Switzerland, the Bobbie Burns country in Scotland, Mammoth Cave in Kentucky, Yellow Stone National Park, or other places of interest. In this course you are to be the tourists and I am to be the guide.

Requirements for the Trip

There are several requirements for becoming a member of any tourist group. Some of these requirements are, good health and physical strength, suitable clothing, ability to follow directions, ability to get along with other people, ability to put the welfare of the group before his own pleasure and welfare, and respect for the rights of other members of the group. It is very desirable that the tourist shall have read about the sights which he will see. Besides this he must have a well filled pocketbook and be willing to spend his money for the trip and for his com-

fort and welfare enroute. Many of the essentials which were not previously provided may be purchased at various points on the journey.

There are likewise several requirements for entrance to this class and for taking this journey through algebra. All of the requirements mentioned above for the sight-seeing trip are essential save one, namely the well filled pocketbook. Health, physical strength, suitable clothing, ability to follow directions, ability to respect the rights of others, ability to get along with others are all essential. Besides these, every member of this group must be able to read, write, and have at least a fair knowledge of the fundamentals of arithmetic. We can dispense with the well filled pocketbook because the people of the state provide a system of free schools for all boys and girls of the state. There is, however, a substitute requirement for the well filled pocketbook. This is a well filled head. Furthermore, you must be willing to spend of your gray matter freely for your improvement and advancement. Just as the tourist who starts on a journey without having all the requirements may often purchase them at some station on the trip, so you who do not now have all of the requirements for this course, may be able to secure them as we go along provided you are willing to spend mental effort freely to get them. There is one great contrast between the tourist who spends of his money and you who spend mental energy, that is the more the tourist spends the less he has, while the more mental energy you spend the more you have to spend.

Reasons for Taking a Trip

Whenever one takes a journey into a strange country his curiosity is naturally aroused. Many questions arise such as: "What shall I see?" "Is the trip worth the time, effort and money that it will cost?" and so on. How do you determine the answers to these questions? Largely by the comments of those who have taken the trip. Thousands upon thousands of boys and girls have taken this trip and it has been found so profitable and desirable that it is now one of the prescribed courses for practically every pupil who passes through high school.

Nature of This Trip

As a high school student I took this journey myself and since that time have been the guide of many groups of boys and girls.

Because of this experience perhaps I should give you some little notion about it and tell you very briefly about some of the things you should look for. It is perhaps more like arithmetic than any other course you have taken in school. However, it is different from arithmetic. Like arithmetic, it treats of numbers, but not always in the same sense in which you are accustomed to think of numbers. So far in your experience you have thought of numbers as represented by the Arabic numerals, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, and so on. For certain purposes, we want to use not only the numbers indicated above but in addition it is very convenient at times to have the number scale extend to the left of zero just as far as the above numbers extend to the right of it. In playing certain games in which the players are required to pay penalties for failure to conform to certain rules of the game, or because of failure to make as many points as have been bid are said to be set, that is their score is decreased by as much as they bid. At the beginning of the game when a player's score is zero if he is set his score is actually less than zero. Sometimes we say "he is 5 in the hole." It seems to me better to say that his score is —5. This means that he has to make 5 points before he has a score of 0. If you look at a thermometer you will see that the scale is numbered from 100° down to 32° and on down to 0° and then down to 30° or 40° below 0°. In certain sections of the country it is not uncommon to find the mercury 10°, 15°, or 20° below zero. In such a case it is common to say the temperature is —10, —15, or —20 degrees. In business when a man's liabilities are greater than his assets we say his financial standing for example is —$10,000, meaning that he has debts amounting to $10,000 and nothing with which to pay them. Other illustrations might be used but these should suffice to show you the need for negative numbers. These negative numbers make it possible to subtract any number from any other number, even a larger number from a smaller one.

Another device peculiar to algebra is the use of letters of the alphabet to represent numbers. There are many problems, the solution of which becomes much easier when this is done. This is true particularly because of another tool which we use in algebra which you used very little or not at all in arithmetic. This is the equation, which is a statement that two expressions are equal. Both the use of letters as numbers and the use of the equation may be illustrated by the solution of the following prob-

lem. I do not expect you to fully understand the solution to this problem, but you can readily see from the proof that I solved it correctly and with very little work.

The Lesson

A base ball nine has won 19 games out of 36 games played. If after this it should win 3/4 of the games played how many games would it need to play to bring its average of games won up to 66 2/3 per cent.?

In algebra we would solve the above problem as follows:

Let x = the number of games that would have to be played.

$3/4x$ = the number of these x games that would be won.

$x + 36$ = total number of games played.

$3/4x + 19$ = total number of games won.

$$\frac{3/4x + 19}{x + 36} = 0.66\ 2/3.$$

$$3/4x + 19 = 66\ 2/3x + 24.00.$$

$$3/4x - 2/3x = 24 - 19$$

$$1/12x = 5$$

$$x = 60$$

Proof

$36 + 60 = 96$ Total games played.

$19 + 3/4$ of $60 = 64$ Total games won.

Number of games won divided by total number of games played equals per cent. won.

$64 \div 96 = 0.66\ 2/3 = 66\ 2/3$ per cent.

It is evident from the foregoing that a problem which would be difficult to solve by arithmetic is easily solved by the use of letters and the equation.

In many kinds of work such as engineering, physics, chemistry, machine shop work, carpentry and plumbing it is necessary that we use the formula. These formulas make problems which are otherwise very hard, relatively easy. To use just a simple illustration: We say the interest is equal to the principal times the rate times the time. The formula is $i = prt$. It is easy to imagine this formula as an abbreviation of the above rule. So almost every rule that has been devised for solving certain types of problems has been translated into a formula which in most cases is simpler and more readily understood than the rule. Because of this fact it is well for every boy and girl to become familiar with algebra. (Of course if you are so fortunate as to go to college you will need algebra in your college work.)

When a party sets out on any trip there are usually dangers that beset them. The guide may point out these dangers, but he cannot keep the tourists from heedlessly walking into them.

Likewise your guide may point out the pitfalls in algebra, but he cannot keep you from falling into them, if you do not heed his warning.

One warning which should perhaps be given at this time is this: unless you see and understand the meaning of each day's journey as we go along you cannot understand the next day's trip. Let me implore you therefore to see and understand the work of each day through careful study.

Let us now turn to the first pages of our textbook which is to be our handbook and your guide when I am not with you.

SUBSCRIBERS NOTICE!

The date of expiration of subscriptions is now being printed on the outside wrapper of every issue of THE MATHEMATICS TEACHER in order that our subscribers may always have before them the month in which their subscription expires. If members of the Council will renew their subscriptions promptly it will save us a great deal of time and energy.

The magazine is now being printed on a very slender financial margin, as you may know, it being the lowest priced magazine of its kind in the country. If we are to continue publication at the present rate and at the same time improve the general appearance and contents of the TEACHER, it will be necessary for all concerned to help cut down unnecessary expenses as much as possible.

It is the policy of the Council to keep the magazine in reach of every classroom teacher of mathematics in America. To do this the expenses connected with publication must be kept at a minimum.

THE EDITOR.

A NEAR TRAGEDY *

By FLORENCE BROOKS MILLER

Fairmount Junior High School, Cleveland, Ohio

Act I

Time: The present.

Place: The realm of Mathematics.

(Father Mathematics is seated upon his throne which is on a raised platform. At his feet sits Arithmetic.)

Father Mathematics: Humanity is certainly working at cross purposes. While making use of me constantly in new inventions such as the radio, the airplane, television, and the like, we hear rumors that some people are belittling us, even neglecting some of my children.

* A play worked up for presentation by the Junior Mathematics Club of Fairmount Junior High School at the close of the school year. Though the Sponsor of the Club, Mrs. Florence Brooks Miller, for the main part wrote the play she was influenced greatly by criticisms and suggestions of the club members.

In the first act the stage was made to appear dignified with draperies, the only furniture being the raised throne upon which Father Mathematics sat. All of the characters with the exception of Mr. Spoiler, the human being, were dressed in flowing robes which had some symbol that designated the branch of mathematics being represented. A crown with the name of the branch of mathematics was also worn. Colors which blended well were chosen in working up the costumes. Father Mathematics wore a white robe, with a heavy white cord around the waist, a gold crown on which was printed in black the word **MATHEMATICS**. He had a long white beard which gave him a venerable appearance.

The characters in the second act were dressed in modern attire as the different parts suggested. In the first scene of the second act the stage was made to look like a dining room with the table set for breakfast. The second scene required very little change to make it look like a living room. The various electrical articles such as an electric fan, a telephone, a lamp are conspicuous first because of their uselessness and later because of their very sudden transformation into usefulness. Behind the scene a motor and several bells attached to it added to the telephone bell and the appearance of the light of the lamp and the running electric fan. All of the electrical appliances were attached to one switch so that at the crucial moment a button was pressed and all started to work at the same time.

Page (Entering and bowing to Father Mathematics): Father Mathematics, your son Demonstrative Geometry is hurrying to see you.

Father Mathematics: Have him enter.

Demonstrative Geometry: Father! It is true! I am not to be given a chance in the High Schools. Only those pupils who choose to study me need do so. How can a boy know whether he would like me or not without being given a chance to become acquainted with me. Isn't there a possibility that he might like me even if he did not care for my brother Arithmetic or sister Algebra?

Father Mathematics: There, there, son. They will have to learn by experience. They will have to find out your importance. You may be neglected for a time and have your feelings terribly hurt but finally humanity will see that you are needed. Don't worry!

Page: Intuitive Geometry enters, Sir.

(All greet her.)

Intuitive Geometry: (Entering, addressing her father and then her brother Demonstrative Geometry) Good evening Father. Demonstrative, where did you run away to this morning?

Demonstrative Geometry: I went to hear my fate. I am to be neglected.

Intuitive Geometry: Not for long though. As the children of humanity study me more they will learn to like me better. They do like me. You see I am more on their thinking level than you are. (Walks over to Demonstrative Geometry and they continue talking in an undertone.)

Page: Here come your daughters, Trigonometry and Algebra, Father Mathematics.

(As they enter all exchange greetings with them.)

Trigonometry: Why! What does this mean? A sob party? What is the matter with Demonstrative Geometry?

Demonstrative Geometry: It is easy for you two to be cheerful these days. You are becoming more popular. I am losing the position I held so long.

Trigonometry: There was a time when only college people studied me, but now I am being introduced to quite young boys and girls.

Algebra: I, too, am made use of by even seventh graders to help them solve their problems.

Page: Father Mathematics, your children, Analytical Geometry and Conic Sections, and Calculus are approaching.

Father Mathematics: Have them come in.

(They enter and all exchange greetings.)

Father Mathematics: Good evening children. What news have you?

Calculus: The citizens are gathering in groups. We overheard disturbing talk.

Conic Sections: Yes, I heard them say such things as these: "My boy thinks Mathematics is too hard." "I don't see why my daughter has to study Algebra." "My girl will never need to know Geometry."

Analytical Geometry: I heard a man say, "Let's do away with Mathematics."

Page: Father Mathematics, your children, Astronomy and Music are without.

Father Mathematics: Tell them to come in here.

(As they enter those already there move naturally around forming groups.)

Astronomy: What is this! A family reunion?

Father Mathematics: Yes, and it is never complete without you and Music, Astronomy.

Music: I heard a girl say that she loved Music but hated Mathematics! (Laughing.) She didn't realize that I am of the Mathematics family.

Demonstrative Geometry: A lot of people think Arithmetic is all there is to Mathematics. Father, how did Arithmetic become so well known?

Father Mathematics: When people began to ask such questions as, "How much?" "How many?" "How long?" they started to count. Then Arithmetic was born. Very shortly after that measuring became a necessity and Geometry arrived upon the scene. I mean Intuitive Geometry. One of the greatest activities of man is that of buying and selling. My son Arithmetic is needed to figure costs and settle accounts.

Arithmetic: In games, too, I am used. Everyone wants to know how to keep score.

Page: Father Mathematics, a human is here to see you.

Father Mathematics: Have him enter.

(All show polite curiosity as the man, Mr. Spoiler, enters.)

Father Mathematics: Good evening Sir!

Mr. Spoiler: Mathematics, altogether too much importance has been given to your family. If people want to study you, well and good. We cannot see why all pupils in our schools have to study your children other than Arithmetic.

(Arithmetic looks around very pompously.)

Father Mathematics: Do you think that you can get along without us?

Mr. Spoiler: Most of us can. If I can, I don't see why the rest cannot.

Father Mathematics: My son Arithmetic is very much in demand but do you use him in adding the long columns of figures in your business?

Mr. Spoiler: No, of course not. I have an adding machine which is more accurate and more rapid, too.

Father Mathematics: Those who made that great convenience for you made use of their knowledge of branches of Mathematics other than Arithmetic, in its construction.

Mr. Spoiler: Not everyone needs to know how to make the machinery which we use. Let those who like to do it, do it.

Father Mathematics: How can you people tell who would like us until all have a chance to make our acquaintance? How would you choose the people who would enjoy knowing about and using us?

Mr. Spoiler: Well, I don't know about that.

Father Mathematics: Have you thought what it would mean to the world to do away with us, or even to limit an understanding of us?

Mr. Spoiler: We don't want to do away with your son Arithmetic, but Algebra and the other members of your family are too difficult for our children to understand.

(Arithmetic looks proudly around while the others look downcast.)

Father Mathematics: Would your wife be satisfied to go back to the use of a broom instead of an electric cleaner to clean her house? Would you like to see the work on the building constructions, bridges, street paving, and so on,

cease? Do you enjoy your radio? Do you ever use the telephone?

Mr. Spoiler: What has all that to do with the question at hand?

Father Mathematics: My children to whom you object play a very important part in the many inventions which make living easier and pleasanter. Think of the many people who must understand how to set up and how to follow instructions in building, in the laying out of properties, the making of roads, tunnels, bridges, to say nothing of automobiles and the many electrical devices upon which you have come to depend. I haven't mentioned the cultural value of my family. With the elevation of your race comes an appreciation of the arts which are very dependent upon Mathematics.

Mr. Spoiler: You have almost convinced me, but I fear it is too late! I hear my fellow citizens coming to overpower you.

(A mob enters crying out, "We don't want any more of you! Down with Mathematics! Away with you!" They bind Father Mathematics and his children.)

(The clock strikes ten as the curtain is drawn.)

Act II

Scene I

Time: The following morning.

Place: Dining room in the home of Dr. Curem.

(Breakfast table is set and Mrs. Curem is busy about it when Dr. Curem enters.)

Dr. Curem: Haven't Mary and John come down yet?

Mrs. Curem: No! I called them and expect them soon.

Dr. Curem: (Looking at his watch and then holding it up to his ear) Why! My watch has stopped!

Mrs. Curem: The kitchen clock stopped at ten last night. I can't understand it because I distinctly remember winding it.

Dr. Curem: That is the time my watch stopped, too.

Mary: (Entering hurriedly) My watch stopped at ten o'clock! What time is it?

Dr. Curem: Well! What does this mean? (Looking at the clock in the room.) That stopped at ten, too.

John: (Entering leisurely) Hello folks!

Mary: John, what time have you?

John: (Looking at his wrist watch) Ten o'clock! (Laughing.) Let's go to bed again. Or is it ten A.M.? I must have forgotten to wind my watch.

Mrs. Curem: I'll call Western Union. (Goes to the phone. Central does not answer. Tries clicking the receiver. Finally gives it up.)

Dr. Curem: Well John, run over to Browns and ask them for the correct time. (John runs out.) Let's sit down and eat. It must be late. (They sit down at the table.)

Mrs. Curem: I guessed at timing the eggs. I hope they are cooked to your liking.

John: (Running in breathless.) They said all their clocks and watches stopped at ten! What do you know about that?

Mary: How wierd!

Mrs. Curem: The cars don't seem to be running. I haven't seen one go by this morning.

John: (Looking out of the window) There goes Mr. Elliott walking

Dr. Curem: Walking! He never walks. When his car is out of order he calls a taxi. Well, I must hurry. Come John, if you are to ride with me.

(Exit Dr. Curem and John.)

Mrs. Curem: (Calling after them) Telephone us the correct time. (Then turning to her daughter who is still sitting at the table) I didn't want to worry Daddy so kept quiet about my experiences this morning. I tried to weigh some sugar and the scales would not work. They would not register. I tried to use the electric sweeper and it was out of order. I went down to the laundry and put some soiled clothes in the washer and there seemed to be no electricity. Why! Here comes Daddy and John! What can be the matter?

(Enter Dr. Curem and John.)

Dr. Curem: The car won't start. We'll have to take the street car. I don't see how I'll call on my patients to-day without the auto.

Mrs. Curem: The streetcars aren't running.

(A rap at the door. Mary goes to the door and lets Mr. Todd in.)

Mr. Todd: Good morning Mrs. Curem. Good morning Doctor. Your bell evidently doesn't ring so I rapped.

Mrs. Curem: More strange things have happened this morning, Mr. Todd.

Mr. Todd: At our house, too. Even our clocks and watches have stopped, and the strange thing about it is that they all stopped at the same time.

All: At ten o'clock!

Curtain

ACT II

Scene II

Time: Evening of the same day.

Place: Livingroom in the home of Dr. Curem.

(Dr. and Mrs. Curem, John and Mary are sitting by candle light.)

Dr. Curem: How I miss the newspaper! This has been a hectic day!

Mrs. Curem: I have come to the conclusion that we had better go back to living as the primitive people did. I can't use any of the mechanical devices. The phone is of no use. Here we sit by candle light. No electricity!

John: You are fortunate to be at home. You have no idea what it has been like at school. No one knew what time it was. We were in history and English classes two hours or more at a time. We had no mathematics at all, no science, no mechanical drawing. It was fierce!

Mary: The funniest thing happened at our school. There we were all studying history when the mathematics teacher stuck her head in the door and said to Miss Myers, "Are we going or coming?" She said the children had forgotten all they had ever known and I overheard her add that she herself could think of nothing to help them.

(A rap at the door. Mary lets Mr. Saveyourmoney in.)

Mr. Saveyourmoney: Good evening Mrs. Curem. Good evening Doctor. Mary. John. (They all exchange greetings.)

Mr. Saveyourmoney: I feel so strange! At the bank to-day I felt as if I were in a world of confusion. There was nothing that went right. Some of the tellers had to be taken to the hospital. They said they felt unbalanced. They could not count money nor add columns.

Dr. Curem: More people have come to me with similar complaints. It evidently is a disease of the mind. We doctors are to have a meeting to-night to exchange ideas.

(A rap at the door. Mary lets in Mr. Buildahouse.)

Mr. Buildahouse: Doctor, this is more than a friendly call. May I see you in your office?

Dr. Curem: Certainly, Mr. Buildahouse. Come right in this way.

(Exit the two men.)

Mr. Saveyourmoney: Poor fellow! I bet I know how he feels!

Mary: (Hiding a yawn.) I think I'll go to bed.

Mrs. Curem: Yes dear. Do go.

(Exit Mary.)

(Dr. Curem and Mr. Buildahouse enter.)

Dr. Curem: The same complaint. Practically the same symptoms but experienced along different lines of thinking.

Mr. Buildahouse: Misery loves company!

(A rap at the door. John lets Mr. Buyfromme in.)

(All exchange greetings.)

Mr. Buyfromme: Doctor, Doctor! I thought I would never get here. The cars aren't running. I can't get my own car to run and I have walked five miles carrying this lantern to light my way.

Dr. Curem: Take this chair Mr. Buyfromme. Mother, would you please get a glass of water for this poor man?

Mrs. Curem: Certainly. (Hurrying from the room.)

Mr. Buyfromme: Thank you. I wish you had been in my store to-day. I feel as if I am losing my mind. We had to bring in oil lamps and lanterns because it was so dark. All my clerks came and asked me questions about costs and measures. They seemed to have forgotten all they ever knew. The funny thing about it was that I couldn't answer them! (Enter Mrs. Curem with the glass of water.) Thank you Mrs. Curem. There were not as many people in the store as usual because of the difficulty in getting down town. My clerks were late. That is most of them were. One came way too early. No one knows whether the money received or the change given were correct or not. Oh, it was terrible!

John: I wonder if we are the only community that is doing without electricity?

Mr. Buyfromme: A man came into my store this afternoon who had driven from Norwood and he said every village he came through was afflicted. We get no sound over the radio.

(A rap at the door. John lets in Mrs. Cutandsew and her daughter Mabel.)

Mrs. Cutandsew: Good evening Dr. Curem. I am Mrs. Cutandsew.

Dr. Curem: Good evening Mrs. Cutandsew.

Mr. Buyfromme: I guess I'll start back home. (Starts for the door.)

Dr. Curem: I'll see you tomorrow, old man.

Mr. Buyfromme: Perhaps. (Exit.)

Mrs. Cutandsew: Doctor I want to talk with you about myself. I seem to have forgotten certain things, others are perfectly clear in my mind. I am a dressmaker and to-day I haven't been able to use my charts to cut out a dress. I can't measure. All day I have accomplished nothing. I sent Mabel to the store and she had an awful time. Tell Doctor about it, Mabel.

Mabel: Mother gave me some money to buy some sugar. The storekeeper didn't know how much sugar to give me. He said the scales didn't work. And he didn't know how much money to give back to me either. Everyone in the store was out of sorts.

Dr. Curem: We are all experiencing a very strange condition. I don't understand it. To-night we doctors are to have a conference and we hope to find out something. Come in tomorrow to see me.

Mrs. Cutandsew: Very well Doctor. Come Mabel. (Exit.)

(A rap at the door. Mr. Spoiler, the dissatisfied citizen is let in by John.)

Mr. Spoiler: Good evening Doctor. My name is Spoiler. I have a great weight on my mind.

Dr. Curem: What is it, Mr. Spoiler.

Mr. Spoiler: Something happened last night! I went too far! My eyes are opened now to the consequences of what I did! Oh! Why did I do it?

John: He's gone entirely crazy.

Mrs. Curem: John, don't talk that way. The poor man!

Dr. Curem: Tell me about it, Mr. Spoiler.

Mr. Spoiler: Well, I would have told no one this morning, but after what I have experienced to-day I'll tell anyone who will listen. (All pay close attention.) For a long time I have been hearing of the difficulties of mathematics. My children have complained and said it was too hard. Other parents told me that their children were suffering because of it and we decided to do away with mathematics. To make a long story short, a crowd of us bound Mathematics last night at ten o'clock.

All: At ten o'clock!

Dr. Curem: Ah! Light is dawning!

John: Well! Clocks do depend upon mathematics don't they?

Mrs. Curem: No wonder I couldn't weigh or measure things!

Mr. Saveyourmoney: Who could expect anything but confusion in a bank without mathematics?

Dr. Curem: Poor Mr. Buyfromme! He thought he was unbalanced. So much of his business at the store depends upon mathematics, measuring goods, making prices, counting change and so forth.

John: Electrical things! How the lack of mathematics cripples the world!

Dr. Curem: (To Mr. Spoiler) Could you take us to the place where the deed was done?

Mr. Spoiler: Yes come with me.

(All go out together excepting Mrs. Curem.)

(Mary enters with a kimona and bedroom slippers on.)

Mary: (Yawning) Have they all gone?

Mrs. Curem: Yes Mary. They have gone to release Mathematics.

Mary: (Putting her arms around her mother) Poor Mother!

Mrs. Curem: No, Mary. I am not unbalanced. Let me tell you what has happened. A certain Mr. Spoiler was here and informed us that he and several dissatisfied citizens had bound Father Mathematics and his children last night at ten o'clock. Naturally when Mathematics was bound, all law and order, all mechanical devices, all measuring and computations vanished.

Mary: Who ever heard of such a thing? Can they unbind them?

Mrs. Curem: They have gone to do so now.

(Lights suddenly come on. An electric sweeper starts running. A telephone bell rings. An electric fan whirls. A motor is heard. The clock strikes.)

Mary: (Looking out of the window) The cars are running! Mathematics is free! *Curtain*

"IF"

A Play in Two Acts Showing the Importance of Mathematics

By RUTH L. SNYDER

Class 1929 Lower Merion High School

Ardmore, Pennsylvania

Act I

Alice, a high school student, sits bent over a table studying. The room is a typical modern living room.

Alice: And cosine squared plus sec $2x - \frac{3}{4}$ the tangent over the cotangent times the cosecant equals 435.27. Now this comes to (consults book) .00125. Goodness, how I hate logs. (Works silently for a bit.) $\frac{13}{17}$ of 49! Why can't they give us even numbers? And do you add or subtract? (Walks across room and looks at notebook, which is on the telephone stand.) Goodness! You divide. And here's a mistake. That makes this five and this (erasing) three and this two. At last! (Again consults book.) What? It isn't right? I've worked two *hours* on it, and now it isn't right. (She stamps her foot, and throws the book across the room.)

A quietly dressed lady enters, and stands looking on.

Alice (finally noticing her): Why . . . hello! Who are you?

Lady: I'm your fairy godmother, my dear. I'm rather in a hurry, but I can grant you one wish before I go.

Alice: I can have anything I want? (Her eye falls on the mathematics book.)

Lady: Anything at all.

Alice: Then I want all math to disappear from the earth.

Lady: You're quite sure that's what you want?

Alice: Yes!

Lady: Very well. Your wish is granted. Good afternoon. (Disappears.)

Alice: Oh, how wonderful! (Tears up paper she has been working on.) Pensively: No more mathematics . . . just

think! Great! She picks up a list and goes to telephone placed on the other side of the stage.

Alice: I must call up the grocer and get these things for supper. Hello? I want Ardmore . . . give me Ardmore. (Aside.) Why, there are no numbers! How disconcerting! (To the phone.) Whom do I want? . . . you may remember? Why, I'd like the grocery—Smith's grocery, please. Yes. I want the one at no.—the one at no.— . . . the one on Elm avenue. (Looks at watch, then shakes it.) And, operator, could you give me the correct time, please? My watch seems to have stopped. What? No time? (Aside.) I guess that's right. Imagine having no time! Hello? Is this Mr. Smith? Will you please send this order to Mr. Jones' house at no. . . . Mr. Jones' house on Maple Avenue, please? Yes, right away. What! What will I pay you with? Why, Mr. Smith! I'm sure father has always been very prompt about paying your bills at the end of the month! No money? Why . . . why . . . I . . . That *is* so. But I think you can trust us. We've been customers of yours for a long time . . . You will? Thank you. Perhaps at the end of a couple of weeks you can go down to father's store and pick out a piece of furniture to square the bill. Yes? Well, I want bread, some potatoes, some beans, some cans of soup—vegetable, tomato, and consomme, and . . . how many? Why, I really couldn't tell you. There are no numbers . . . and I want some sugar—I guess you'll just have to send a bagful. Then I want some bacon, a few eggs—not very many, please, and quite a lot of those chocolate cakes. No, nothing else. Yes. Thank you.

Sits at phone, thinking, as curtain falls. Shakes watch, vigorously, once or twice.

Act II

In the same room, the next day, a group of boys and girls are gathered.

Alice: Was that the bell? (Goes to door, and brings back a boy and girl—general cries of Hello, and Well, look what's here, etc.) Well, I *never* had such a day before! I had to walk to school—there weren't any trolleys, and I'm

sure that I was late . . . but I may have been early, for all anybody knows. And of course there weren't any classes; no time, you know. Bells rang, but how were we 'sposed to know which class it was? Personally, I spent all morning in mathematics, because I knew there wouldn't be any work. I don't know anybody who went to English, do you?

Bob: I did. The mathematics room was so crowded I couldn't get in.

Alice: Well, I bet you were lonesome! Then I was late for lunch, and most everything was gone. I was so bored doing nothing all morning I went to Physics class in the afternoon—at least I suppose it was afternoon—it was after lunch. Well, I never heard so much guessing in my life! They guessed this was so, and "'sposed" that was true. They couldn't prove a thing!

John: Huh! That's nothing! You should have been in Chemistry. We hardly could do anything. There weren't any formulas, or any rules, or any more theories. We tried some experiments, and something happened, but we couldn't figure out what it was. Something combined with something, but it might have been any of several different things.

Emma: Well, what about art, and music? I went to a concert last night, and the conductor couldn't keep time—it was amusing to watch the poor man, and the harmony was *rare!* He'd go like this (slow wave of the hand) and then like this (quick wave). And the instruments were without dimensions, and such funny things you never saw—nor heard, either. And think of the futuristic art—it depends entirely upon angles and curves—why, there won't be any art, except maybe color.

Lewis: There isn't even that. They have to mix colors in proportion.

Emma: Why didn't I think of that? And dressmakers are losing their jobs, too. They can't get patterns, nor make correct measurements.

James: Well, my father says he doesn't know what he's going to do. You know, he's one of the engineers on that big bridge, and he says all building has stopped everywhere, absolutely; tunnels, bridges, railroads, buildings . . . they

can't do a thing without a lot of figuring. Well, that's thrown an awful lot of people out of work—builders, makers of building material, and people who depended on them to sell things to shopkeepers. Father says he doesn't know what's going to happen. Everything's tottering. I guess our whole civilization is based on mathematics.

Susan: My father's a manufacturer, and he's had to shut down his whole factory. He makes iron, and he says it's no use making a thing unless you have the formula to make it by. And he not only has no means of knowing how much of anything to use but he doesn't have any accounts, nor statistics, nor anything. He couldn't pay the men, even. But my brother's even more worried than that.

Alice: The one that's going to get married?

Susan: Yes. His fianceé was going to make the curtains for their new home, but now she can't, because she can't measure the amount of goods. They'll just have to send over batches of curtains and shades from the store until they find some that fit. And he can't buy his rugs, and . . .

Emma: That brother of yours had better hurry up and buy his linoleum, too, because they can't make any more without geometric designs. He won't be able to get wall-paper, either.

Louise: My father's worried, too. He had some stock in a mine, and now, of course, Wall Street's stopped absolutely—it depends entirely on calculation—and he doesn't know what's going to happen to the stock. The mine's closed; it can't run any longer—might cave in any minute, and it couldn't transport the coal if it was running—railroads need time, and accurate recordings of position, and all sorts of mathematics. Well, no one wants the stock—it's no good to anybody. Just think of all the money lost forever, tied up in stocks and bonds!

John: Well, you can have the money. It's nothing but dirty pieces of paper and silver, now. I'd rather have a horse. The farmers are lucky—they still have something—their fields, and crops.

Lewis: Huh—do you think so? They're not so well off, either; how are they to know when to plant things? The calendar relied upon some very intricate mathematics; they may plant things too late, now, or too early, and get

caught by the frost either way. And all the new farm machinery's not so much good any more. It'll run until it needs repairs, and then there won't be any new parts to replace old ones with. Everything's topsy-turvy! I used to hate mathematics, myself, but I certainly wish it were back. Whatever could have happened to it?

(Alice blushes, but no one notices it.)

Louise: No one seems to know. All the mathematics just disappeared, all at once, and that's all anybody knows about it. I've got to go. My sister is due back from Europe, and we don't know what's happened to her—ships can't steer at all without mathematics. I hope there's some news. . . . Well, goodbye, everybody. (Leaves.)

Alice: Would you care to play cards?

Susan: How can we? We can't count. We don't even have any amusements left. I've got to go. (Visitors all drift out casually—giving the usual excuses—dinner, etc.)

Alice is left; she goes slowly across the room, finally sitting down with her head in her hands. The fairy godmother appears.

Lady: Good afternoon, my dear. I was in a hurry yesterday, so I came back now to give you another wish you may have. What do you want?

Alice (slowly): Oh dear! I guess I'll have to wish mathematics back again.

Curtain.

SUBSCRIBERS PLEASE NOTE

We are caused a great deal of inconvenience and so are some of the former officials of the National Council because people continue to write to those who used to edit and manage the Council's publications instead of to the present officials.

All communications relating to the magazine, THE MATHEMATICS TEACHER, should be addressed to

THE MATHEMATICS TEACHER
525 West 125th Street
New York City

COLLEGE ENTRANCE REQUIREMENTS IN GEOMETRY

By PROFESSOR DUNHAM JACKSON

University of Minnesota
Minneapolis, Minnesota

A proposal has been made to the College Entrance Examination Board that it should modify its requirements so as to bring about the more extensive introduction of courses including an appreciable amount of solid geometry in the first year of geometry, in place of a part of the plane geometry ordinarily taught. In response to a request from the Board, a committee has been appointed by the Mathematical Association of America and the National Council of Teachers of Mathematics to discuss the feasibility of the proposal. The membership of the committee is as follows:

Miss Gertrude E. Allen, University High School, Oakland, Cal.
C. M. Austin, High School, Oak Park, Ill.
Ralph Beatley, Graduate School of Education, Harvard University, Cambridge, Mass.
Walter F. Downey, English High School, Boston, Mass.
Mrs. Elizabeth L. Hall, East High School, Rochester, N. Y.
J. O. Hassler, University of Oklahoma, Norman, Okla.
Dunham Jackson (Chairman), University of Minnesota, Minneapolis, Minn.
C. N. Moore, University of Cincinnati, Cincinnati, Ohio
W. D. Reeve, Teachers College, Columbia University, New York, N. Y.
Edwin W. Schreiber, Western Illinois State Teachers College, Macomb, Ill.

The primary function of this committee is not to draw up detailed recommendations for the proposed new requirement, but to discover if there is sufficient interest in the project on the part of colleges and schools to justify the Board in proceeding with a careful study of it.

The inclusion of any significant amount of new material in the first-year course clearly implies the elimination of much that has been regarded as of genuine importance. The question is not

whether the existing course can absorb the additional material, which ultimately may be proposed, but whether it can clear a place for it without sacrificing its own essential character.

Those who believe in the existing courses in geometry at all will agree that the pupil ought to carry away with him:

An adequately comprehensive knowledge of geometric ideas, facts, and processes.

An intimate acquaintance with the nature of deductive reasoning, as applied not only to detached items of argument, but also to the sustained building up of an extensive and coherent logical structure;

Familiarity with the independent use of deductive reasoning through the study of substantial "originals";

Some facility in the application of geometrical knowledge in the world of experience.

It is suggested that it may be found possible to preserve these essentials, with considerably more liberal recognition than has been customary hitherto of the principle that an elementary course need not aim at the final articulation of *all* the facts that it embraces into a single logical framework. It may be possible to arrive at a readjustment of emphasis which will admit some of the important ideas of three-dimensional geometry in the first year, and at the same time bring geometry closer to the rest of mathematics and to the other sciences.

While the present committee does not aim at a detailed working out of the project, it invites discussion by individuals and groups of teachers who may be interested. Communications may be addressed to the chairman or to any member of the committee.

NEWS NOTES

Mathematics at Exeter *

"I was taught here to learn." These words and those of the parent who says, "You taught my son to work," contain the tribute perhaps most commonly paid to the Academy. What is meant is that the drive of the place has led the boy to acquire habits of industry

* This article appeared in the September issue of the Bulletin of the Phillips Exeter Academy. It was written by Mr. Harry C. Barber the president of the National Council of Teachers of Mathematics. We think it will be of interest to our readers.—*Editor.*

and persistence, to assume responsibility for his own success, and to refuse to be beaten by hard tasks. Realizing that he had to compete in order to get in to the school and that he had to struggle in order to remain, he readily accepted the hardwork tradition of a school in which "the student bears the laboring oar."

This drive and the boy's response to it is a characteristic of the Academy which must be understood before the work of any department can adequately be discussed. Two other general characteristics should be mentioned. This is a school for able boys. Modern education increases its efficiency by grouping students according to their ability. At Exeter this separation automatically occurs because boys from the lower ranges of ability are very generally excluded. Here the intelligent boy competes with his equals, and methods of instruction can be adapted to his capacities and needs. And finally there is success in examinations for college entrance. Few failures occur and high averages are maintained by those who pass their courses in the Academy.

It is obvious that these three characteristics of the Academy constitute a challenge to each department. The pace must be maintained. It is, on the other hand, just as obvious that they do not in themselves constitute a sufficient scholastic aim. None of the departments would be content with plans which did not contemplate more than this. All of them are accustomed to say, in effect, that they expect not only to send boys on into college but to send them there with eager intellectual interests and with the capacity and will to grow. Now it is clear that such an achievement is not entirely a matter of specific training, of information, and of petty "skills"; rather it has to do with attitude of mind. It may well be that this is the true measure of success; that excellence in a school lies in the attitudes of mind which it creates.

The more one ponders the problem the more convincing this conclusion becomes. The attitude is the thing. Success and failure depend upon it. It determines all the adjustments of our lives. One educated man is a criminal, one an irresponsible idler, and a third a benefactor; it is all a matter of attitude of mind.

This conclusion is not only sound but far-reaching in its effects, for when we accept as the measure of our success the mental attitudes which we develop in our students, it follows that every method and device and the whole spirit of the institution will be determined with regard to the attitude of mind which they produce. With this conclusion in mind we turn to the discussion of what the department of mathematics hopes to do.

I

Many of the traditions of the department are associated with Wentworth, of masterful personality, whose textbooks made his name a household word throughout the country. He helped to shape school algebra into a well defined unit of work. He was a leader in the reform that put originals into geometry. How well Mr. Francis carried on the work is not so widely known, but it is clearly indicated by the opinions expressed by his students who have been long enough out of

school to see their education in perspective. Two alumni, one himself a mathematics teacher of long standing, and the other a business man well versed in mathematics, have told me recently not only of their admiration for the man and of his devotion to the Academy, but of the unusually sane and stimulating view he gave them of mathematics, its nature and its purposes. Such voluntary tribute is not easily won. These two teachers have left us the task of continuing the progress they began.

Wentworth was himself the whole department. Now we pool the wisdom and experience of seven men cooperating for the improvement of instruction. To bring about this mutual study and cooperation was one of the problems resulting from the growth of the school, and it is interesting to see that the traditional democracy of the Exeter faculty lays the right foundation for the necessary free interchange of ideas. In other words, during the present period of growth of American schools there is no more important movement than that looking to the improvement of instruction through cooperative endeavor and discussion on the part of the teachers, and to this plan of cooperation the free give and take of the faculty meeting and the general traditions of the school are notably well adapted.

In the effort to maintain past excellence and to adapt it to modern times we know that we must move ahead. We recognize the difficulty of weighing the old against the new, of making changes without loss. We know too that our subject is so old and has been so long a part of the school curriculum that some would think no changes in it need be made. Yet changes are being made, so many in fact that we now commonly hear the phrase, "the new mathematics." There is of course no new mathematics in the field of the secondary school, but two things are taking place: the same two that took place in Wentworth's day: I. The usual periodic revision of aims, material, and method; II. The perennial struggle of good teachers, each in his day and generation, to resist the bad trends and to support the good. The preface of a mathematics text of one hundred years ago boasts that pounds, shillings, and pence are replaced by decimal coinage in all its problems, and that its students are taught to think and to depend upon themselves. Prefaces to-day make not dissimilar claims.

We have in mind both this revision and this upward struggle, and they may be said to give color to much that is distinctive in mathematics at Exeter. It so happens that school mathematics falls easy prey to certain ills. They are suggested by such words as rote, routine, drill for drill's sake, empty symbols, and meagre aims. This is particularly true, as has been pointed out again and again, in schools which prepare for external examinations. The mere mastery of a process so that it can be reproduced tends to be magnified over the intellectual curiosity which may be aroused, the methods of investigation which may be taught, and the direction which may be given to thinking by the proper approach to these processes and to the purposes which they serve. Resistance to the commonly recognized belitting tendencies of the mathe-

matics classroom is one of the marks of good teaching. We try to have in mind always that "overhaste to impart mere knowledge defeats itself." Even the so-called cramming school can fix the processes. It is the margin which we get over and above the processes themselves by which we want to stand or fall.

II

This brings us back to our first conclusion, that excellence in a school or in a department lies in the attitudes of mind which it creates. There are many mental attitudes which we may hope to develop. Some of them are suggested in the paragraph above. Perhaps the most interesting and the most important one for our department is that which may be called the scientific attitude of mind. The present age is pre-eminently scientific. It is set apart from all other ages by scientific discovery and invention which are due to the scientific method and the scientific attitude of mind; and thoughtful men are already predicting that the outstanding characteristic of the coming generation is likely to be the application of this method to more and more of man's problems with more and more success. President Eliot wrote, "The nineteenth century brought into the world for the service of education, as well as for the service of industries and government, the new temper of mind called the scientific; and the effects of this new temper or spirit have been nothing less than revolutionary." It is surely true that in order to understand the genius of our own age we must know something of the scientific method of thought. It is probably true that in the preparation of our students for the age in which they are to live we can perform no greater service than to make them familiar with this method and its use. We may even believe that in this complex age the straight, systematic, and unprejudiced thinking that is characteristic of the scientific habit of mind is the necessary foundation for the building in intelligent character.

Now mathematics offers the earliest and best opportunity to acquire the scientific habit of mind. Of course it is true, as the reader is no doubt thinking, that the random study of the subject without regard to methods of approach or to the resulting attitude of mind may completely fail to accomplish this end. No one can safely claim that this is not so, but the opportunity is there, and we believe that if we persistently seek the development of this attitude of mind, and if we use our material intelligently for the purpose, much can be accomplished.

III

One more mental attitude is deserving of our close attention. The question at issue is this: How much do we want the interest of the student in his work? Much, little, or not at all? So long as he does the hard work, need we be concerned about this interest in it? This question is fundamental because it determines so many details of the teaching methods to be used.

Life brings to nearly all of us difficult tasks. We know the value of learning to do hard work. Even the school boy does, at the bottom of his heart. This does not mean, how-

ever, that the secondary school must condemn him to four years of hard, uninteresting labor. Interest in his work does not make him less diligent, less successful, or less happy. Interest does more than anything else to develop that willingness and eagerness of mind which is fertile soil for education. Lack of genuine interest is probably the most common cause of failure.

Whatever can be accomplished by hard work under compulsion, all that and more can be accomplished by hard work impelled by interest. The galley slave bears the laboring oar under the threat of the lash, but the school crew labors just as hard because of interest in the outcome. Both kinds of rowers develop muscle, but the school boys probably develop the better attitude of mind. In life's lower ranges there is the compulsion of fear or hunger, but it is interest which leads to the painting of a great picture or the writing of a poem. It was interest which led Archimedes to double the knowledge of geometry which was handed down to him by Euclid.

This brings us back again to the heart of the matter. We must admit that when we undertake to prepare our students for college in such a way that they will have the desire for more education and the ability and determination to succeed, when we accept their attitude of mind as our major aim, we have an aim which cannot be attained by external pressure alone. The groundwork of success is the interest of the students, and the larger the proportion of them who work hard because of their own interest the better we are succeeding. It is clear, of course, that we are not talking about idle curiosity or mere amusement but about interest of the sort which makes one want to get to work and achieve something.

The scientific attitude and the attitude of interest rather than of yielding to compulsion are not without relation to each other. When the student realizes that "Mathematics underlies our present day civilization in much the same fundamental way that sunshine forms the source of all life and activity on the earth," and when he finds that he is learning to use the scientific method of thought, he is very likely to develop an interest in his daily work. As he progresses through one, two, three, or four years of such study of mathematics he may be expected to show noticeable growth year by year not only in knowledge but also in that understanding and questioning attitude of mind which may properly be called education.

The Mathematics Section of the Northeastern Ohio Teachers Association held its autumn meeting at Cleveland, Ohio, on October 25, 1929, Mr. H. C. Wagner of Warren, Ohio, presided. The program follows:

1. "Bringing the Mathematics Curriculum up to Date." Gertrude Fowler, East High School, Youngstown.
2. "Adjustment of Instruction to Individual Needs." Martha McCready, Salem.
3. "Prize Problems for Prize Pupils." Professor Elbert H. Clarke, Hiram College.

The Mathematics Section of the 84th annual meeting of the New

York State Teachers Association (Southeastern District) gave the following program on Friday afternoon, November first, in Macy Hall, Teachers College, Columbia University, with George W. Miller of Memorial High School, Pelham, presiding:

Address: "Topics in Algebra." May Porter, White Plains.

Address: "Starting Point in Geometric Problems." George W. Miller, Pelham High School.

Address: F. Eugene Seymour, Supervisor of Mathematics, State Education Department.

The following communication has just been received by the editor of the MATHEMATICS TEACHER:

In the October, 1929, MATHEMATICS TEACHER Joseph A. Nyberg shows how the base angles of an isosceles triangle may be proved equal without assuming the possibility of bisecting an angle. He claims that his proof "will give a more logical structure" than the one involving the angle bisector.

It is stated that to bisect an angle, as is done in the ordinary textbook demonstration, requires for its proof the theorem that two triangles are congruent if the three sides of one are respectively equal to the three sides of the other. Since this congruence theorem is not proved until after the isosceles triangle theorem, the assumption of an angle bisector is defective. With this we agree.

However, Nyberg's proof assumes the possibility of constructing an angle equal to a given angle. This construction can no more be done without the third congruence theorem than can the bisecting of an angle. To prove that the constructed angle is equal to the given angle makes use of the theorem that two triangles are congruent if their three sides are respectively equal. Hence Nyberg in his proof runs into the very same difficulty that he set out to avoid.

It is our opinion after considerable trial that the best approach to geometry can be made, as suggested by Christofferson, by assuming all three congruence theorems at the outset. Then the simple constructions can be done and these will be available when needed.

Sincerely yours,

P. H. NYGAARD

The Mathematics Section of the High School Conference gave the following program at Urbana, Illinois, on November 22, 1929:

Morning Session

"Some Desirable Characteristics in a Modern Plane Geometry Text." Mr. L. E. Mensenkamp, Freeport High School.

"Parallelism between Reasoning in Geometry and Reasoning in other Fields." Mrs. Elsie Parker Johnson, Oak Park and River Forest, Township High School.

Round Table—Suggestions for Conducting the Class Period in Geometry.

Afternoon Session

Business.

Address—"Mathematics and the Public." Professor Herbert E. Slaught, University of Chicago.

Address—"The Cultural Values of Algebra," Professor James Byrnie Shaw, University of Illinois.

NEW BOOKS

Modern Geometry. By ROGER A. JOHNSON. Houghton Mifflin Company, 1929. xiii + 319 pages. $3.50.

"This book deals with the geometry of the triangle and the circle . . . based entirely on the elementary plane geometry of Euclid or its modern equivalent. . . . The student who is familiar with high school mathematics and with the language of trigonometry is well qualified to reap the full benefit of a course in this subject. . . ."

It is gratifying to record a preface that actually lives up to its promise. In the particular instance of "Modern Geometry" the body of the book more than fulfils the pledge. Here we have Plane Geometry continued, widened, deepened, enriched, and adorned. To the student, or teacher, of mathematics who regards Euclidean geometry as a dead or dying branch of his science, "Modern Geometry" may prove to be a surprise, a shock, a revelation, or, at the very least, a potent argument against despoiling —be it ever so gently—high school geometry. To the lover of geometry, who has regretted ever since his high school days that he had but one year to give to it, "Modern Geometry" will furnish happy reminiscenes, new joys, new opportunities, a new challenge.

Champions of plane geometry may well take heart. If plane geometry is to be saved, better teaching will do it. The prospects for better teaching grow brighter with two excellent "college" geometries in the field.[1]

The title page of Professor Johnson's book bears the added legend: "An Elementary Treatise on the Geometry of the Triangle and the Circle." Furthermore, throughout the book, the author addresses himself to "the reader" rather than the student. "Modern Geometry" is thus more than a mere textbook. Even for Euclidean geometry, than which no more "continuous" subject exists, "Modern Geometry" patently exhibits a continuity of material and a unity in presentation rarely discoverable in textbooks. Thus, "the reader," provided he carefully follows in the footsteps of the author, will get along merrily without the benefit of extraneous instruction. The "proved" content is generally presented in such nicely graded steps that he who writes as he reads on will doubtless "get it all."

The greatest intrinsic obstacle that "the reader" will need to overcome is the rather perplexing horde of more or less unfamiliar terms. Many an innocent theorem seems imposing and formidable because of the (unfortunately unavoidable) terminology employed. As, for example, the last corollary in Section 494: "The circumcircle, the Brocard circle, the Lemoine line, and the isodynamic points belong to a coaxal system [the Schoute coaxal system], orthogonal to the circles of Apollonius." We therefore warn the [new] reader to pay

[1] The other is *College Geometry*, by N. Altshiller-Court. Johnson Publishing Company, Richmond, Virginia, 1925.

due homage to the DEFINITION and, in general, to adhere rather closely to the sequence laid down by the author.

To most students of geometry the triangle and the circle are merely the triangle and the circle.[2] Yet over 500 theorems, many of exquisite elegance, great power, and intrinsic beauty, form the material of the "Elementary Treatise on the Geometry of the Triangle and the Circle." And the greater wonder of it all is that "the logical sequence is clear throughout, so that the reader will seldom be puzzled by any real difficulty."

The shortcomings of the book are rather minor and are mentioned chiefly to complete the review. Many pages are "too full": where theorems are given as exercises following each other in close succession, page monotony, albeit in italics, is the consequence. Ideally, diagrams omitted to make "the reader" "draw his own" is commendable textbook pedagogy, but in practice, and oft repeated practice at that, it is a sufficient, but not a necessary, strain. The book is remarkably free of serious errors. Slips or typographical errors occur: (1) page 39, Section 58e, should read:

$$t^2 = \overline{DC}^2 - r^2 = \frac{K^4 - 2K + 1}{4(K^2 - 1)^2} \cdot \overline{AB}^2 = \tfrac{1}{4}\overline{AB}^2;$$

[2] A primrose by a river's brim
A yellow primrose was to him
And it was nothing more.

(2) page 105, Section 142, following "Combining these proportions." the complete set of equalities should be given as:

$$\frac{\overline{OA}}{\overline{OB}} = \left[\frac{\overline{OA}_1}{\overline{OB}_1}\right] = \frac{\overline{OB'} - \overline{OA}_1}{\overline{OA'} - \overline{OB}_1} = \left[\frac{\overline{A_1B'}}{\overline{B_1A'}}\right] = \frac{\overline{AP}}{\overline{BQ}};$$

(3) page 139, Section 197, the word *concurrent*, in the third line of the theorem, should be replaced by *collinear;*

(4) page 221, Section 355, the third line of the theorem should read" . . . the triangle *is a minimum* is the isogonic center R.

May the sales of the book prove so large as to enable the publishers to sell it at a considerably lower price, if not to the general public, then at least to teachers of geometry. To the latter group, particularly, the book is strongly recommended. At the suggested lower price we should feel free to add that no teacher of geometry can afford to remain ignorant of the contents of Professor Johnson's enriched source book, textbook, and treatise on the geometry of the triangle and the circle, rather inadequately described by its shorter title: "Modern Geometry."

JOSEPH SEIDLIN.

ALFRED UNIVERSITY,
ALFRED, N. Y.

OFFICIAL NOTICE

As Secretary of the National Council of Teachers of Mathematics I hereby officially announce the annual election of certain officers of the National Council, said election to take place at Atlantic City, New Jersey, on Friday, Feb. 21, 1930. Article III, Section 7 of the By-Laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity through announcement in the official journal to suggest by mail for the guidance of the Directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the Secretary of the Board of Directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the Board of which the Secretary shall be chairman. The election shall be by mail or in person and shall close on the day of the annual meeting."

For your consideration the Secretary has compiled a register of all persons who have served the National Council in an official capacity. These names appear on the following page.

The officers to be elected are: (1) President, 1930–1932, (2) Vice-President, 1930–1932, (3) Three Directors, 1930–1933. As a convenience to the Secretary he requests that you indicate your suggestions on a 3 x 5 card, thus:

President	(over)
Vice-President	Name of member sending suggestions .....
Three Directors	Address .....
.....	Date .....
.....	
(over)	

Kindly mail suggestions immediately to

EDWIN W. SCHREIBER, *Secretary,*
434 W. Adams St.,
Macomb, Ill.

Register

1. Allen, Gertrude .....Dr. 23, 24, 25
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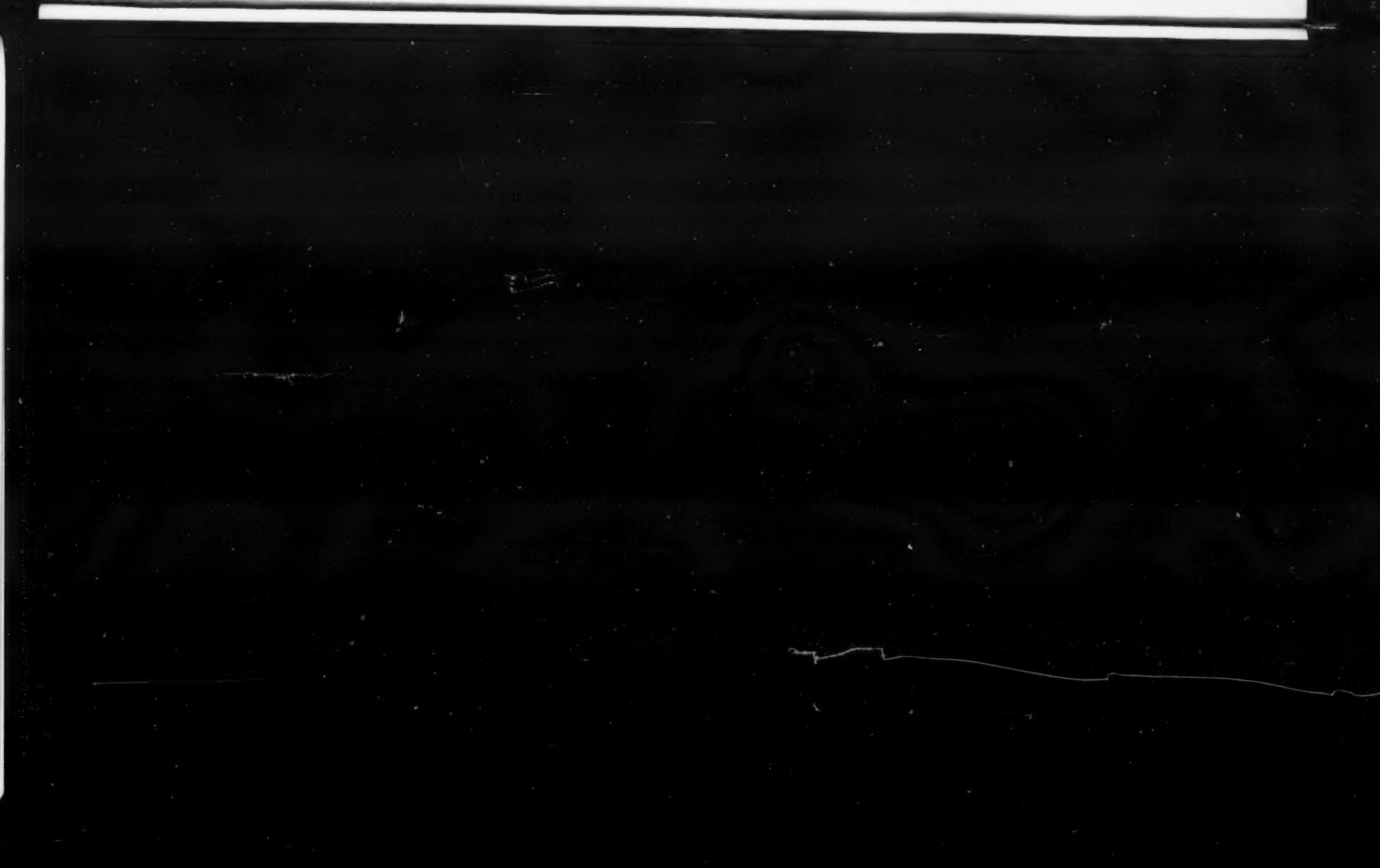

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